LECTURE NOTES

ON

THEORY OF STRUCTURE

FOR DIPLOMA IN CIVIL ENGINEERING

4th SEMESTER AS PER SCTE&VT SYLLABUS

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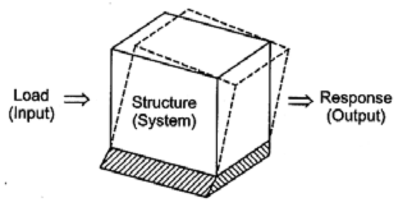
Topic 1 : Structural Analysis

STATIC AND KINEMATIC INDETERMINANCY

Structural engineering is a field engineering dealing with the analysis and design of structures that support or resist loads. Structural engineering is usually considered a specialty within civil engineering, but it can also be studied in its own right. Structural engineering theory is based upon physical laws and empirical knowledge of the structural performance of different materials and geometries. Structural engineering design utilizes a number of simple structural elements to build complex structural systems. A structure may be defined as an assemblage of load bearing elements in a construction.

Structural Analysis
Structural analysis is the application of solid mechanics to predict the response (in terms of force and displacements) of a given structure subjected to specified loads.

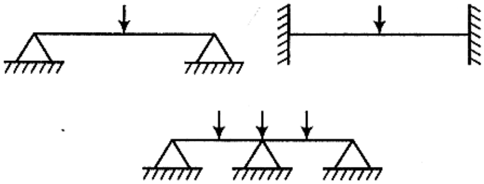


3D Structural view of an element

Structural Elements
Structural elements are used in structural analysis to split a complex structure into simple elements. Within a structure, an element cannot be broken down (decomposed) into parts of different kinds {e.g., beam or column). *Structural elements can be linear, surface or volumes as given below.*
Line element → Beams, columns, truss, frame, cable.
Surface element → Slabs, shear walls etc.

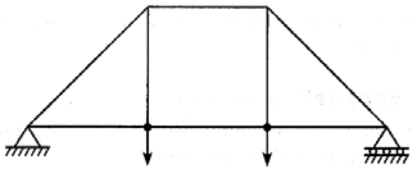
Planar Structures
When all the line elements of skeletal structure lie in a single plane and the loading is also in this plane, the structure is called planar structure otherwise, it is called a space structure.
Some example of the Planar Structures are

Beams
Simply supported, fixed or continuous.



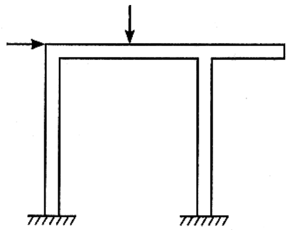
Simply supported beams

Plane Trusses
In plane trusses, all the members lie on a plane, while the loads carried by the truss, are only concentrated forces that act on the joints and lie on the same plane.



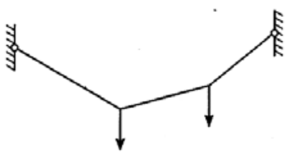
Developed plane trusses

Plane Frames
In the case of plane frame, all the members lie in the same plane and are interconnected by rigid joints.



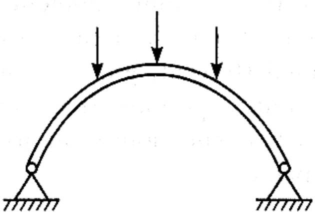
Plane frames

Cables
Cables carry applied load and develop mostly tensile stresses. Loads applied through hangers. Cables near the end supporting structures experience bending moments and shear forces.



Load acting on cables

Arches
Arches carry applied loads and develop mainly in-phase compressive stresses; three hinged, two hinged and fixed arches.



Load acting on arches

Joints
Various elements in a skeletal structure are interconnected at joint.
Every joint serves two important functions

Kinematic function At joint, different connecting parts should be displaced identically (rotate or translate).

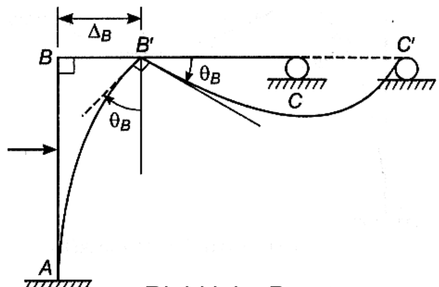
Static function Joint should provide appropriate transmission of internal forces (axial, shear, bending moment, twisting) from one connecting member to other.

Types of Joints

Joints are divided into various types as given below

Rigid Joints At rigid joint, there can be no relative rotation or translation occur between connecting member.

Thus, this joint fully transmits all types of internal forces. Joint itself may move or translate.



Rigid joint B

In above figure, joint B is rigid, under the application of load frame deflects as shown, with rigid joint B rotating θ_B and translates Δ_B. The rigidity of joint B ensures that the included angle between connecting member (AB and BC) remains unchanged (right angle in this case).

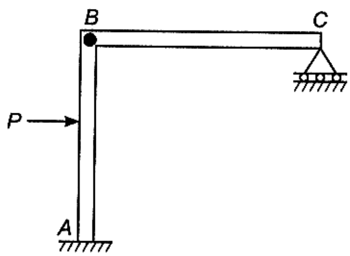
Both members (AB and BC) will undergo same clockwise rotation θ_B.

Pinned Joints The pinned joint permits free relative rotation between interconnecting members, while ensuring that the translation is the same. Freedom of rotation, ensures the bending moment is zero at pin joint. Transmission of axial and shear forces is possible. e.g., Natural pinned (hinged) joints in our own physical bodies shoulder joint (permits free movements in all directions), elbow and knee joints (limits the freedom of rotation in one plane).

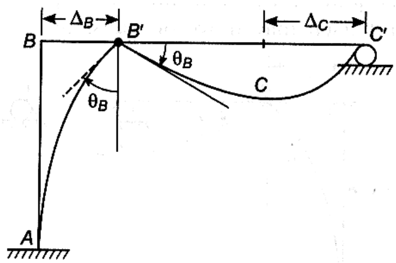
Frames

Frames are assemblies of elements that resist force through a stiffness of the beam-to-column joints or which contain additional diagonal elements to brace motion against collapse.

In given frame, providing a pinned joint at B, internal angle between AB and BC is no longer required to remain a right angle. Column AB behaves like a vertical cantilever, with no flexural participation from connecting beam B.



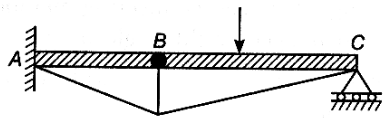
(a) Simple frame ABC



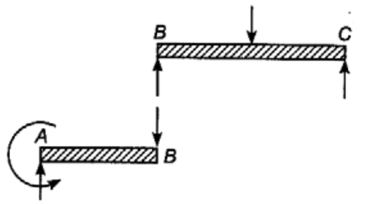
(b) Rigid joint B in frame ABC

Rigid joint B in frame ABC

Beams
Beams are slender members used for supporting transverse loading.



(a) Internal hinge at B in a beam



(b) Load transfer mechanism

FBD

Shear force diagram of a beams

Pinned joint, provided to interconnect two beams is called internal hinge. The hinge (internal pin) releases moment at B in above beam. Load transmission from AB to BC takes place via shear force.

Supports (for a planar structure)
The different types of supports generally used to balance a beam are given below

Fixed Support (or built in)
All three degrees of freedom arrested, i.e., for plane x-y no translation as well as rotation.

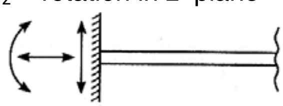
No translation i.e., $\Delta_x = 0$, $\Delta_y = 0$

No rotation i.e., $\theta_z = 0$

where, Δ_x = translation in x–plane

Δ_y = translation in y–plane

θ_z = rotation in z–plane

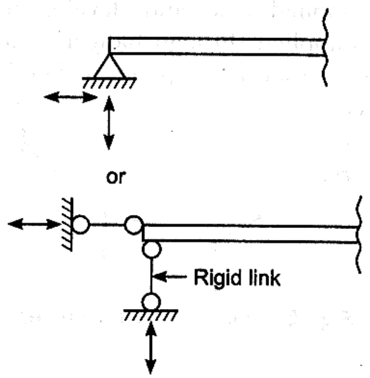


Fixed support

(i.e., three restraints)

Hinged Support (for pinned)
No translation, but rotation is possible (i.e., two restrains)

$\Delta_x = 0,\ \Delta_y = 0,\ \theta_z \neq 0$

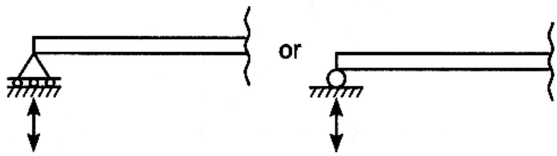


Hinged support

Roller Support
Free to translate in horizontal direction only.

i.e., $\Delta_y = 0$ (one restraint)

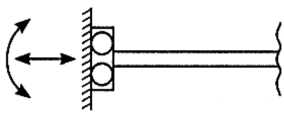
$\Delta_x,\ \theta_z \neq 0$



Roller support

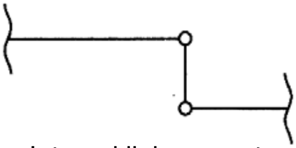
Guided Fixed Support
Only translation in vertical direction is possible.

$\Delta_y \neq 0$



(two restraints)
Guided support

Internal Link
It is a short bar with pin at each end. A link is capable of transmitting moment as well as horizontal reaction.



Internal link support

Thus, it provides two additional equations of equilibrium.

$\Sigma M = 0$

and $\Sigma H = 0$

In statically determinate structures, reactions and internal forces can be determinated solely from free-body diagrams and equations of equilibrium. While in statically indeterminate structures, reactions and internal forces cannot be found by statics alone.

Statically Determinate Structures
Structures that can be analysed with the help of equations of static equilibrium alone i.e., All the forces in the structure can be strictly determined from the static equilibrium equations.

Equations of Static Equilibrium
For spaced structure (i.e. in three dimensions)

$\Sigma M_x = 0$, $\Sigma M_Y = 0$, $\Sigma M_Z = 0$

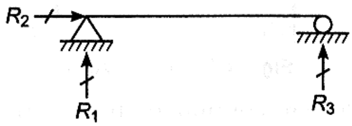
$\Sigma F_X = 0$, $\Sigma F_Y = 0$, $\Sigma F_Z = 0$

For planar structure (i.e. having all members in one plane)

$\Sigma F_X = 0$, $\Sigma F_Y = 0$, $\Sigma M_0 = 0$ (X–Y plane)

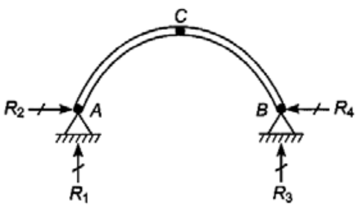
where, M_X = moment in x-plane
M_Y = moment in y-plane
M_Z = moment in z-plane
F_X = force acting on x-plane
F_Y = force acting on y-plane
F_Z = force acting on z-plane

e.g., **Simply supported beam**



Total number of unknowns = 3
Total available equations of equilibrium = 3
So, we can find out R_1, R_2, R_3

Three hinged arch



Reactions on points A and B while C as fixed point

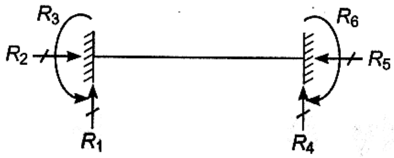
Total number of unknowns = 4
Total available equations of static equilibrium = 3
Extra equation of equilibrium = 1(M_C = 0)
where, M_C = Moment of point C.
Thus, we can find out R_1, R_2, R_3, R_4
Other examples are cantilever beam, overhang beam, a suspension cable.

Statically Indeterminate Structure
These are structures which cannot be fully analysed by using the equations of static equilibrium alone. The number of unknown forces are more than the number of

equilibrium equations. For complete analysis additional equations based on conditions of compatibility are used.

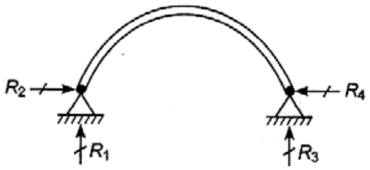
e.g.,

Fixed beam Total number of unknown forces = 6
Available equations of static equilibrium = 3



Fixed Beam

Two hinged arch Total number of unknown forces = 4
Available equations of static equilibrium = 3



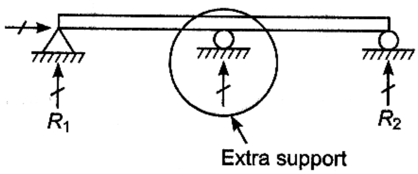
Hinged Arch

Other examples are continuous beams, fixed end frame etc.

Degree of Static Indeterminacy

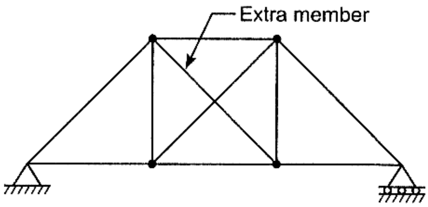
Degree of static indeterminacy also known as degree of redundancy. The word redundant means extra, not needed etc. Thus, when we provide more supports than minimum required for external stability, we make the structure externally redundant.

e.g.,



Simple support beam with extra support

When we provide more internal constraints that, minimum required to make the system rigid we make the structure internally redundant.

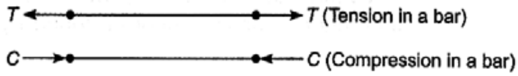


Bending moment diagram

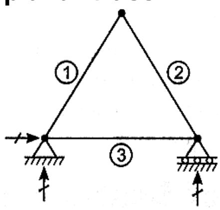
Therefore, extra support reactions are unknown forces in externally redundant system and are called degree of external redundancy or external indeterminacy (D_{Se}).

Extra member forces are unknowns in internally redundant system and are called degree of internal redundancy or internal indeterminacy (D_{si}). Overall static indeterminacy (D_S) = D_{Se} + D_{Si}.

Determinations of D_{Se} and D_{Si}
The external indeterminacy (D_{Se}) and internal indeterminacy (D_{Si}) can be calculated in different section by considering the following calculations.
Trusses A truss is composed of bar elements. A bar element can have only one unknown force either compression or tension, (i.e., only axial force)



e.g.,
Simply supported planar truss

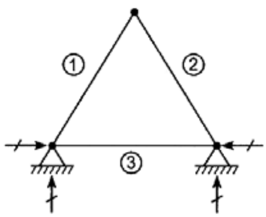


Simply supported planar truss

Unknown support reaction = 3
Unknown member forces = 3 {Number of members)
Total unknown forces acting on truss = 3 + 3 = 6
Now, at each joint there are equations of equilibrium

$$\Sigma F_X = 0, \Sigma F_Y = 0$$

i.e., two equilibrium equation per joint
For given truss, total number of joints = 3
Thus, total equilibrium equations = 3 x 2 = 6
Hence, above structure is statically determinate (D_S = 0)
e.g.,



Unknown reactions = 4
Unknown member forces = 3
Total unknown forces = 7
Equation of equilibrium per joint = 2
Number of joints = 3
Total equilibrium equations = 2 x 3 = 6
Therefore, degree of static indeterminacy = 7 − 6 = 1

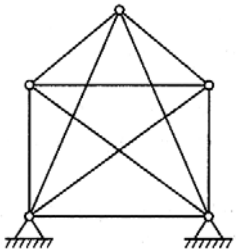
General Expressions (Planar truss)
- Let, r = number of unknown support reactions
  m be number of members i.e. (unknown member forces)
  ∴ Total number of unknown forces = m + r
- Let, J be the number of joints in truss.
  The number of equilibrium equations = 2J
  as (2 equilibrium equations per joint)

Thus, degree of static indeterminacy

= Unknown forces – Equilibrium equations

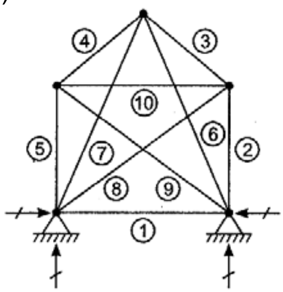
D_S = (m + r – 2J)

Example 1. What is the degree of static indeterminacy of the plane structure as shown in the figure below?



(A) 3 (B) 4 (C) 5 (D) 6

Soln. (B)



Number of unknown support reactions = 4

Number of unknown member forces = 10

∴ Total unknown forces = 4 + 10 = 14

Number of joints = 5

Available equations per joint = 2

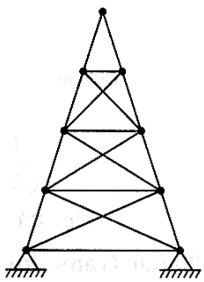
{$\Sigma F_X = 0$, $\Sigma F_Y = 0$}

∴ Total equilibrium equations = 5 x 2 = 10

Thus, D_S = Total unknown forces – Total equilibrium equations

= 14 – 10 = 4

Example 2. What is the total degree of static indeterminacy of the triangular planar truss shown in the figure?

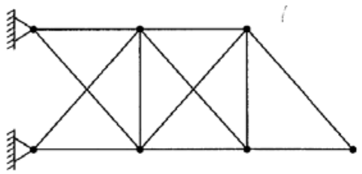


(A) 2 (B) 4 (C) 5 (D) 6

Soln. (B)
Number of unknown support reactions, r = 4
Number of unknown member forces, m = 18
∴ Total unknown forces, m + r = 18 + 4
= 22
Number of joints, J = 9
∴ Available equations of equilibrium = 2J
= 18
Therefore, degree of static indeterminacy

$$D_S = (m + r) - 2J = 22 - 18 = 4$$

Example 3. What is the total degree of indeterminacy of cantilever planar truss shown in the figure?



(A) 2 (B) 3 (C) 4 (D) 5

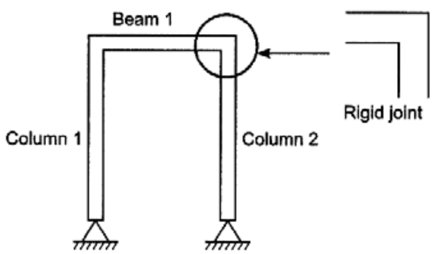
Soln. (A)

m = 12
r = 4
J = 7

$$\therefore D_S = (m + r) - 2J = (12 + 4) - 2 \times 7 = 2$$

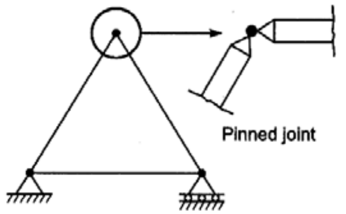
Planar Frame
It is structure made up of the combination beam and column generally with each other by rigid joints.

Kinds of Planar Frame
Planar Frame for Columns



Planar frame of two columns

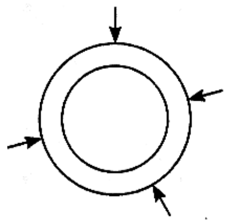
Planar frame for a truss



Truss in triangular form

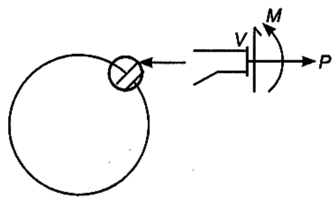
Ring Concept

Let us take a general plane frame member. It may be a generalized ring and is subjected to loads and because of loads, it deforms. Therefore, internal forces are developed in the ring.



Forces developed ring

Now, these internal forces can be find out by making a cut.

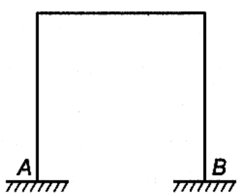


Force make cut ring

A cut release three internal forces shear (V), axial force (P) and bending moment (M) at a section.
Hence, total unknown member forces = 3
Now, applying this concept in planar frame.
Here, we cannot use truss formula for degree of static indeterminacy

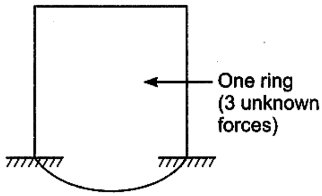
$D_S = (m + r) - 2J$

Make a Planar Frame into a Ring



Planar frame ring

At a fixed end three unknown forces are developed, so ends A and B can treated as cut of a ring i.e.,



Planar frame cut–ring

Hence, degree of static indeterminacy of above frame = 3

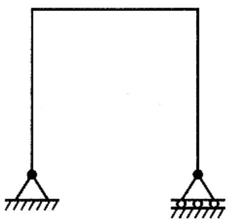
Ring Concept for Beams

For converting a beam into a ring, all supports of the beam have to be made fixed, then subsequently required release are found out.

Degree of static indeterminacy (D_S)

= (3 x Number of rings) – (Number of releases)

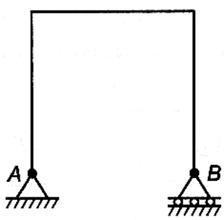
Example 4. What is the degree of static indeterminacy of the planar frame as shown in figure?



(A) 1 (B) 2 (C) 3 (D) 0

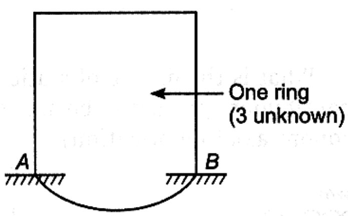
Soln. (D)

To convert the given frame into a ring. End supports A and B should be fixed.



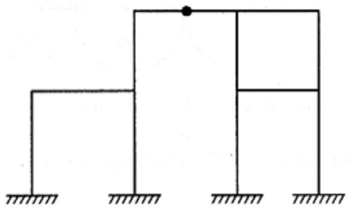
Therefore, D_S = 3 – Number of releases

= 3 – 3 = 0



Original structure has end A hinged (So one release) end B roller. (So two release)

Example 5. What is statical indeterminacy for the frame shown in figure?



(A) 12 (B) 14 (C) 11 (D) 15

Soln. (C)

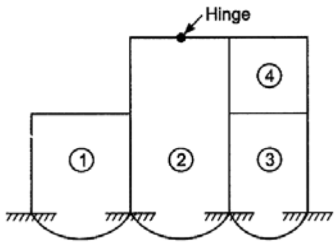
Converting the given framed structure into ring structure; Number of rings = 4

Number of unknown forces per ring = 3

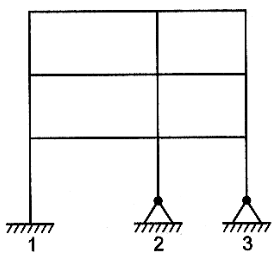
∴ Total unknown forces = 4 x 3 = 12

Number of releases = 1 (hinge)

Thus, D_S = 12 – 1 = 11



Example 6. The total degree of static indeterminacy of the plane frame shown in the given figure is



(A) 18 (B) 16 (C) 14 (D) 13

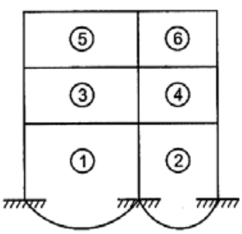
Soln. (B)

Converting the given frame into a ring structure

Number of rings = 6

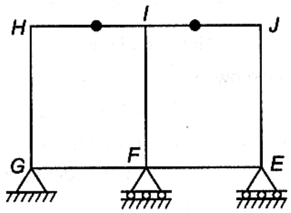
Number of unknown forces per ring = 3

∴ Total unknown forces = 6 x 3 = 18



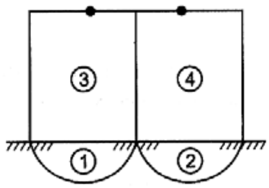
Number of releases = 2 [(each from support (2) and (3)]
Therefore, D_S = Total unknown forces – Number of releases
= 18 – 2 = 16

Example 7. The degree of static indeterminacy of the rigid frame having two internal hinges in the figure below is



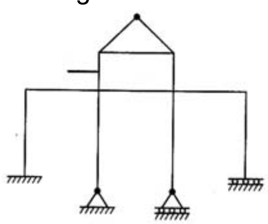
(A) 8 (B) 7 (C) 6 (D) 5

Soln. (D)
Converting the given framed structure into a ring structure



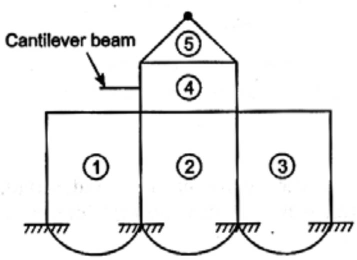
Number of rings = 4
Number of unknown forces per ring = 3
∴ Total unknown forces = 4 x 3 = 12
Number of releases = 2 (internal hinge) + 1 (hinged support G)
+ 4 [roller supports (F) and (E)]
= 7
Therefore, D_S = 12 – 7 = 5

Example 8. The degree static determinacy of the rigid frame as shown in the figure.



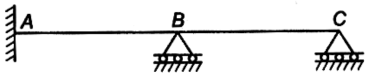
(A) 4 (B) 6 (C) 8 (D) 10

Soln. (D)
Converting the given frame into a ring structure.



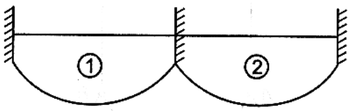
Number of rings = 5
Number of unknown forces per ring = 3
∴ Total unknown forces = 5 x 3 = 15
Number of releases = 1 (internal hinge) + 1 (hinged support) + 2 (roller support) + 1 (fixed roller)
= 5
Therefore, D_S = 15 – 5 = 10
Cantilevered beam should not develop any unknown forces (No ring).

Example 9. The degree of static indeterminacy of the following continuous beam is



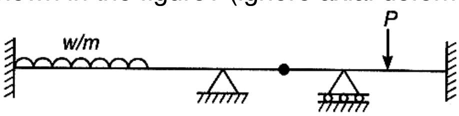
(A) 1 (B) 2 (C) 3 (D) 4

Soln. (B)
Converting the given beam into a ring structure
Total unknown forces = 3 x Number of rings



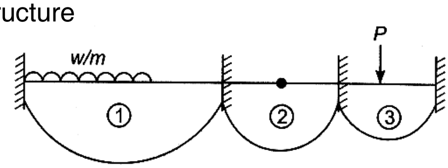
= 3 x 2 = 6
Number of releases = 2 (roller B) + 2 (roller C)
= 4
Thus, D_S = 6 – 4 = 2

Example 10. What is the degree of static indeterminacy in the continuous prismatic beams shown in the figure? (ignore axial deformation)



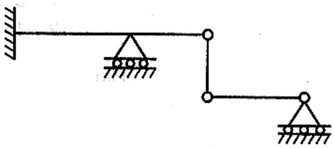
(A) 1 (B) 2 (C) 3 (D) 4

Soln. (D)
Ring structure



Total unknown forces = 3 x Number of rings
= 3 x 3 = 9
Number of releases = 1 (hinged support) + 2 (roller support) + 1 (internal hinge)
= 4
Thus, D_S = 9 – 4 = 5
As loading is vertical we can ignore axial deformation of the beam,
So one more release (axial force) i.e.,
D_S = 4
Unless and until it is specified that axial deformation of beam is ignored. We will not assume it. So, here in above problem [D_S = 4].

Example 11. The degree of static indeterminacy for the beam as shown in figure will be



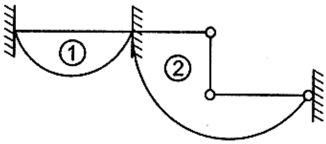
(A) 0 (B) 1 (C) 2 (D) 3

Soln. (B)

An internal link provides two equations of equilibrium i.e.,

Two release ($\Sigma M = 0$, $\Sigma H = 0$)

Ring structure



Total unknown forces = 3 x Number of rings = 3 x 2 = 6

Number of releases = 2 (roller support) + 1 (hinged support) + 2 (internal link) = 5

Therefore, $D_S = 6 - 5 = 1$

Stability of Structure

To ensure the equilibrium of a structure or its member, it is not only necessary to satisfy the equations of equilibrium, but also the members must be held (constrained) by their supports.

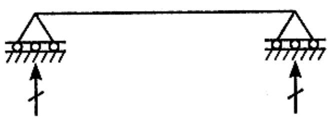
Two situations may occur which should be avoided

Partial Constraints

This is the condition when the structure is not stable then it is free to move horizontally (i.e., the structure is subjected to roller supports). This condition can be avoided when,

Available support reactions < Equation of equilibrium

e.g.,

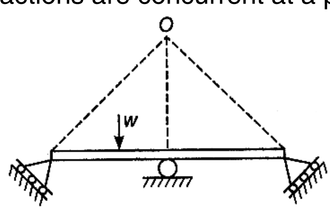


Partial constraints

($\Sigma F_X = 0$) is not satisfied for above structure. Therefore, structure is not stable as it is free to move horizontally.

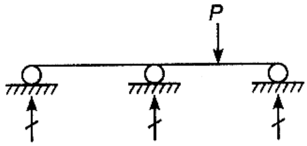
Improper Constraints

All support reactions are concurrent at a point.



Improper constraints

In this case summation of moments about point O will not be zero. (Moment of $w \neq 0$) And we know that for a planar structure summation of moments about all points should be zero. Thus, in above structure rotation will take place about point O {Rotationally unstable}. All support reactions are parallel.



Support section are parallel

If an inclined load is applied in above structure. It will move in horizontal direction (as $\Sigma F_X \neq 0$)

- An unstable structure is avoided whether it is statically determinate or indeterminate.

Degree of Freedom for Structure

When a structure is loaded, specified points on it, called nodes, will undergo unknown displacements. The displacements are referred to as the degrees of freedom for structure.

It is the minimum number of independent coordinates required to define the structure in the displaced configuration, relative to the original configuration.

Kinematic Indeterminacy

It is the total number of degrees of freedom at the various joints in a skeletal structure. To determine the kinematic indeterminacy, we can imagine the structure to consist of a series of members connected to nodes, which are usually located at joints, supports, as the ends of a member, or where the members have a sudden change in cross-section.

The degree of freedom for some individual frame is described below.

Degree of Freedom of Space Frame

- Every joint has 6 degrees of freedom (3 translations and 3 rotations).
- For space truss only 3 degree of freedom (only translation per joint).

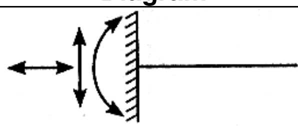
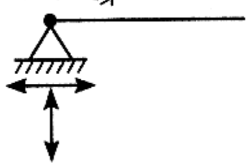
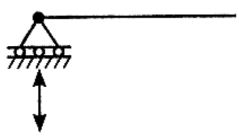
Degree of Freedom for Plane Frame

Every joint has 3 degree of freedoms (2 translation and 1 rotation).

Degree of Freedom for Plane Truss

Only 2 degree of freedom (only translation) per joint.

Degree of Freedom for Various Supports

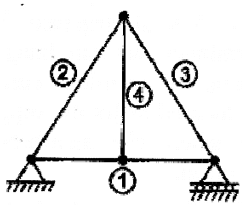
Types of Support	Diagram	DOF	Restrains
(a) Fixed support		Zero	3
(b) Pinned support		1	2
(c) Roller support		2	1
(d) Free end		3	0

Direction of arrows shows that movement is restricted in both right and left directions.

KI of Truss [Planar Truss]
A truss is a structure composed of slender members joined together at their end points. The members common by used in construction consist of wooden struts, metal bars, angles or channels.

- Find out the total number of joints, let it be J.
- Total degrees of freedom = 2J (as two degree of freedom per joint)
- Find out number of restraints at each joint.
- Kinematic Indeterminacy (KI) = Total degree of Freedom − number of restraints

e.g.,

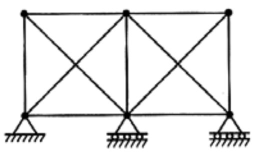


Planar truss

Number of joints = 4
Total degree of freedom = 4 x 2 = 8
Number of restraints = 2 (hinged support) + 1 (roller support)
= 3
KI = 8 − 3 = 5

- **A hinged support** restraints both vertical and horizontal movement.
- **A roller support** restraints only vertical movement.
- **A truss** is always loaded at its joints.

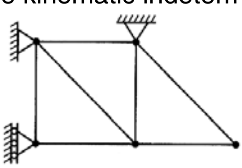
Example 12. For the truss shown in the figure, the kinematic indeterminacy is



(A) 0 (B) 2 (C) 4 (D) 8

Soln. (D)
Number of joints, J = 6
∴ Total degree of freedom = 2 x 6 = 12
Number of restraints
= 2 (hinged support) + 2 (1 from each roller support)
= 4
Therefore, K = 12 − 4 = 8

Example 13. The kinematic indeterminacy of the following truss will be



(A) 3 (B) 4 (C) 5 (D) 6

Soln. (C)
Number of joints, J = 5
Total degree of freedom = 5 x 2 = 10
Number of restraints = 4 (two from each hinged support) + 1 (roller support)
Thus, KI = degrees of freedom – restraints = 10 – 5 = 5

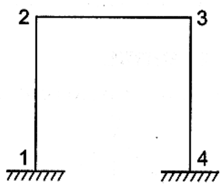
KI of Framed Structure (Planar frame)

The degree of freedom of a structural system is represented by the number of the possible unknown displacements that the different joints of a structures can have.

Let us consider a simple plane frame,

(a) Count number of node (joint) = J

- Each node has three degree of freedom's in a planar frame.

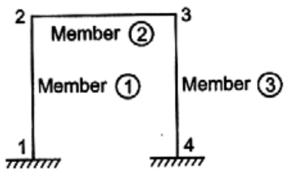


Thus, total number of degree of freedom's
= 3J = 3 x 4 = 12

(b) find out number of restraints at fixed support has all three restraints = 6 (three from each fixed support)

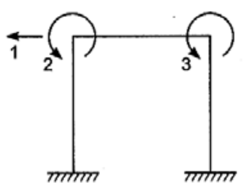
Therefore, KI of frame
= Total degree of freedom – restraints
= 12 – 6 = 6

⇒ Now, a general behaviour of all members of a frame is that they are axially rigid i.e., columns and beams have no axial deformations. These are called constraints.
Thus, KI of frame
= Total degree of freedom – Number of restraints – Number of constraints



Each member has one constraint (axial deformation)

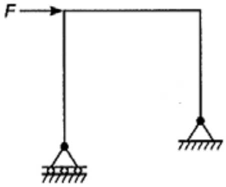
Therefore, KI = (3 x 4) – (3 x 2) – (1 x 3)
= 12 – 6 – 3
KI = 3



Kinematic Indeterminacy of Beam

Same procedure as followed in finding out KI of framed structure.
KI = Total degree of freedom – Number of restraints – Number of constraints

Example 14. Considering beam as axially rigid, the degree of freedom of a plane frame shown in figure below



(A) 9 (B) 8 (C) 7 (D) 6

Soln. (B)

Number of joints = 4

Degree of freedom per joint in a planar frame = 3

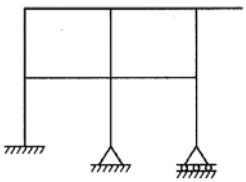
∴ Total degree of freedom = 3 x 4 = 12

Number of restraints = 2 (hinged support) + 1 (roller support)
= 3

Number of constraints = 1 (axial deformation in beam)

Thus, KI = 12 – 3 – 1
= 8

Example 15. For the plane frame with an overhang as shown below, assuming negligible axial deformation, the degree of kinematic indeterminacy, K are



(A) 10 (B) 13 (C) 11 (D) 9

Soln. (B)

Number of joints, J = 10

Total number of degree of freedom = 3 × 10 = 30

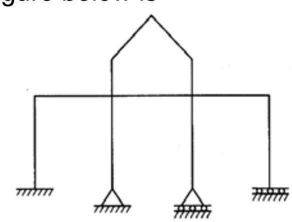
Number of restraints = 3 (fixed support) + 2 (hinged support) + 1 (roller support)

Number of constraints = 11 (one from each member)

Thus, KI = Total degree of freedom – Number of constraints – Number of restraints
= 30 – 11 – 6
= 13

Free cantilever end is also a node.

Example 16. Neglecting axial deformation, the kinematic indeterminacy of the structure shown in the figure below is



(A) 12 (B) 14 (C) 20 (D) 22

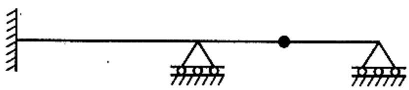
Soln. (B)
Number of joints, J = 11
Total degree of freedom = 3 x 11 = 33
Number of restraints = 3 (fixed support) + 2 (hinged support) + 1 (roller support) + 2 (fixed roller support)
= 8
Number of constraints = 11 (one from each member)
Thus, KI = 33 – 8 – 11
= 14
Fixed roller restraints rotation and vertical movement i.e.,
Number of restraints = 2

Example 17. What is the number of independent degree of freedom (KI) of the two span continuous beam as shown in figure? (ignore axial deformation)

(A) 1 (B) 2 (C) 3 (D) 4

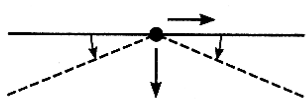
Soln. (B)
Number of joints = 3
Total degree of freedom = 3 x 3 = 9 {three per each joint}
Number of restraints = 1 + 1 {one from each roller support} + 3 {fixed support}
Number of constraints = 2 {one for each span}
Thus, KI = 9 – 5 – 2 = 2

Example 18. The kinematic indeterminacy of the following continuous beam with an internal hinge will be (ignore axial deformation)



(A) 3 (B) 5 (C) 7 (D) 9

Soln. (B)
Number of joints = 3
Degree of freedom per joint = 3
∴ Total degree of freedom = 3 x 3 = 9
An integral hinge has four degree of freedom 2 translation and 2 rotation.



Thus, including hinge, total degree of freedom = 9 + 4 = 13
Number of restraints = 3 (fixed support) + 2 (hinge support) + 1 (roller support)
= 6
Number of constraints = 2 (one from each span)
Therefore, N = 13 – 6 – 2 = 5

METHODS OF STRUCTURAL ANALYSIS

To perform an accurate analysis a structural engineer must determine such information as structural loads, geometry, support conditions and materials properties. The results of such an analysis typically include support reactions, stresses and displacements. This information is then compared to criteria that indicate the conditions of failure. Advanced structural analysis may examine dynamic response, stability and non-linear behavior.

Virtual Work Method (Unit Load Method)
The Virtual Work Method (VWM) is method provides a general means of obtaining the displacement (deflection) and slope (θ) at any point on a structure. The loaded structure may be a **beam, frame** or **truss**.

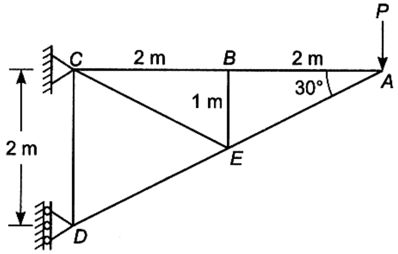
Mathematical Expression of VWM
For equilibrium of a structure subjected to P system of forces then the work done by external loads and work done by internal loads should be equal. i.e.,
External work = Internal work

$$\Sigma P.\ \Delta = \Sigma u.\delta$$

Application of Virtual Work Method for Truss Member
The following steps are involved to solve problems under virtual work method for truss member

Step 1 Remove all the external forces from the truss and apply unit load in the desired direction of Δ.
Step 2 For the corresponding virtual unit load, find out system of virtual internal forces n in all truss members.
Step 3 Apply all real loads and find out system of real internal forces N in all truss members.
Step 4 Use $\Delta = \Sigma\, n \dfrac{NL}{AE}$

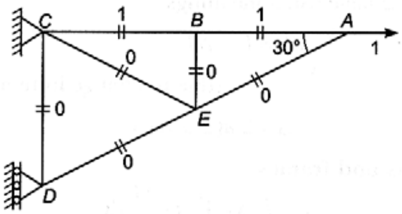
Example 19. For the given truss structure as shown in figure. What will be the horizontal deflection of the joint A? (Take area of each member A and elastic modulus E)



(A) $\sqrt{3}$ P/AE (B) $2\sqrt{3}$ P/AE (C) $4\sqrt{3}$ P/AE (D) zero

Soln. (C)
Step 1 For horizontal deflection at A, apply unit load in horizontal direction.
From analysis of truss,

$$n_{BC} = 0$$



$n_{CE} = 0$

From equilibrium of joint A,

$n_{AE} = 0$

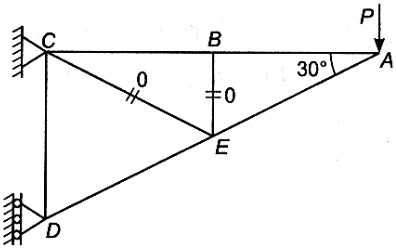
$n_{AB} = 1$

Support D is roller i.e.,

$n_{CD} = 0$

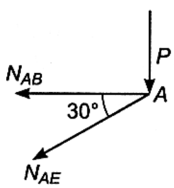
and $n_{BC} = 1$

Step 2



If at a joint three members passes out of which two are collinear, then force in third member is zero provided that there should not be any external force at that joint.

Step 3 Equilibrium of joint A,



$$N_{AE} \sin 30° + P = 0$$

or $N_{AE} = -2P$ (compressive)

$$N_{AE} \cos 30° + N_{AB} = 0$$

or $$N_{AB} = +2P.\frac{\sqrt{3}}{2}$$

$$= +P\sqrt{3}$$

Equilibrium of joint B : $N_{BL} = N_{AB} = \sqrt{3}\,P$

Equilibrium of joint E : $N_{DE} = N_{AE} = -2P$ (compressive)

Equilibrium of joint D : $N_{CD} = -P$ (compressive)

Step 4

Member	n	N	A	L	E	$\frac{nNL}{AE}$
AB	1	$\sqrt{3}P$	A	2	E	$\frac{2\sqrt{3}P}{AE}$
BC	1	$\sqrt{3}P$	A	2	E	$\frac{2\sqrt{3}P}{AE}$
CD	0	–P	A	2	E	0
DE	0	–2P	A	$\sqrt{2}$	E	0
EA	0	–2P	A	$\sqrt{2}$	E	0
CE	0	0	A	$\sqrt{2}$	E	0
BE	0	0	A	1	E	0
					$\Sigma\left\{\frac{nNL}{AE}=\frac{4\sqrt{3}P}{AE}\right\}$	

Horizontal deflection at A = $\frac{4\sqrt{3}P}{AE}$

Application of Virtual Work Method in Beams

- For deflection, apply a unit load in the direction of Δ.

$$\Delta = \int_0^L \frac{mM}{EI}dx$$

- For slope, apply a unit couple at the point of rotation.

$$\theta = \int_0^L \frac{m_\theta . M}{EI}dx$$

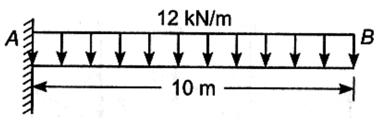
where, m_θ is the virtual internal moment as a f_n of x due to unit couple and EI is the flexural rigidity.

Procedure to Solve Problems under VWM

The following steps are involved to solve the problems under unit load method in beams.

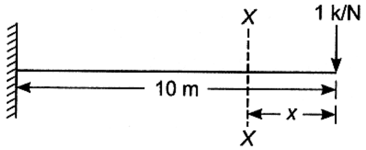
- Remove all external loading from beam, apply unit load at desired location of deflection.
- Find out m as function x.
- Apply all loading (real), find out M as function x.
- Use formula for Δ and θ.

Example 20. For the cantilever beam AB as shown in figure, the vertical deflection at B will be? (take, EI = constant)



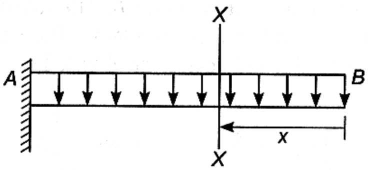
(A) $\frac{5184}{EI}$ (B) $\frac{12384}{EI}$ (C) $\frac{15000}{EI}$ (D) $\frac{31104}{EI}$

Soln. (C)
Step 1



$m_x = -x$

Step 2



$$M_x = -\frac{12x^2}{2}$$

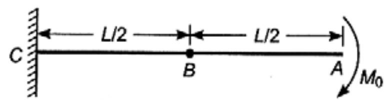
Step 3

$$\Delta_B = \int_0^{10} \frac{(-1x)\left(-\frac{12x^2}{2}\right)}{EI} dx$$

$$= \frac{6}{EI}\left(\frac{x^4}{4}\right)_0^{10}$$

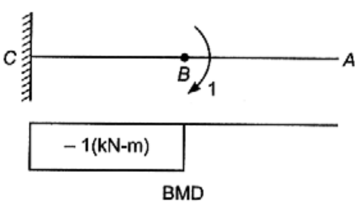
$$= \frac{15000}{EI}$$

Example 21. What is the slope at point B for the cantilever beam AC as shown in the figure? (EI is constant)



(A) zero (B) $\frac{M_0L}{2EI}$ (C) $\frac{M_0L}{EI}$ (D) $\frac{2M_0L}{EI}$

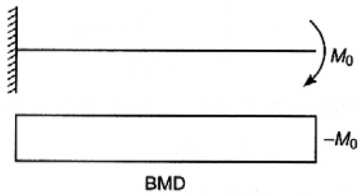
Soln. (B)
Step 1



For $x \in \left(0, \frac{L}{2}\right), m_\theta = 0$

For $x \in \left[\frac{L}{2}, L\right], m_\theta = -1$

Step 2



For $\quad x \in \left[0, \frac{L}{2}\right]$, $M = -M_0$

$\quad x \in \left[\frac{L}{2}, L\right]$, $M = -M_0$

Step 3

$$\theta_B = \int_0^L \frac{m_\theta M}{EI} dx$$

$$= \int_0^{L/2} \frac{0.(-M_0)}{EI} dx + \int_{L/2}^{L} \frac{(-1)(-M_0)\,dx}{EI}$$

$$= \frac{M_0 L}{2EI}$$

Work Done by a Force

If a load w acts on a member and produces deflection δ in its line of action by virtue of its own direction.

Then, work done, $W = \frac{1}{2} w.\delta$ ....(i)

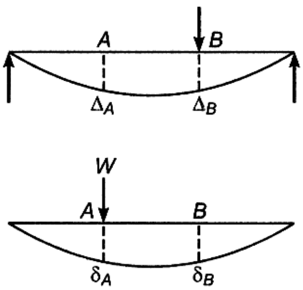
If a load w acts on a member and a deflection y is produced in its line of action due to some other agency. Then, virtual work done by load w is

$$W = w\,.\,y \qquad \text{....(ii)}$$

Maxwell's Reciprocal Theorem

This is also known as **Maxwell-Betti reciprocal theorem**, states that for a linear elastic structure subject to two sets of forces $\{P_i\}$, i = 1....m and $\{Q_j\}$, j = 1, 2, ...n, the work done by the set P through the displacements produced by the set Q is equal to the work done by the set Q. This theorem has applications in structural engineering.

In a structural system (beam or truss) behaving in linear elastic manner. Deflection at any point A due to load w at any other point B is the same deflection at B due to the same load w applied at A.

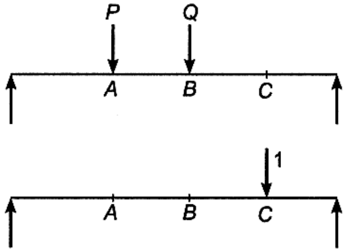


Beam or truss in linear elastic manner

where, Δ_A = displacement at point A

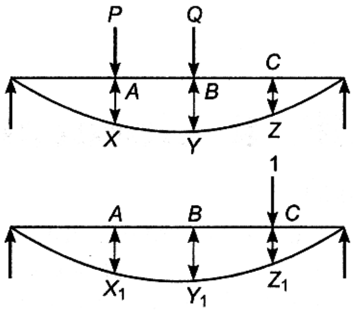
δ_B = deflection at point B

Example 22. In the figure shown below, X, Y, and Z are the deflections under A, B and C due to loads P and Q. X_1, Y_1 and Z_1 are defections under A, B and C due to unit load at C. The deflection Z would be equal to

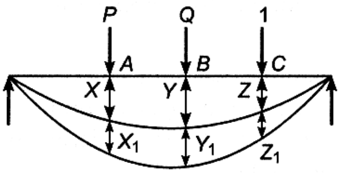


(A) PX + QY (B) PY + QX (C) $PX_1 + QY_1$ (D) $PY_1 + QX_1$

Soln. (C)



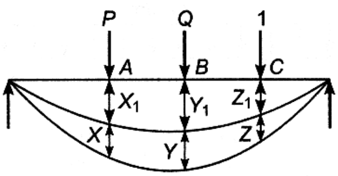
By combining these two cases, let P, Q and 1 acts simultaneously.
[Initially P, Q acts then unit load at C]



Total work done,

$$W_1 = \frac{1}{2}PX + \frac{1}{2}PY + \frac{1}{2}1.Z_1 + P.X_1 + Q.Y_1$$

Now, let unit load acts firstly and then load P and Q.

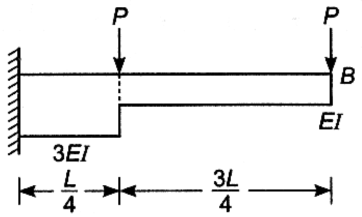


Total work done,

$$W_2 = \frac{1}{2}.1.Z_1 + \frac{1}{2}P.X + \frac{1}{2}Q.Y + 1.Z$$

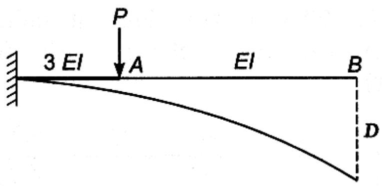
Since, both cases are finally same.
Therefore, $W_1 = W_2$
or $Z = P . X_1 + Q . Y_1$

Example 23. The cantilever beam shown in figure has load P acting at points A and B. The deflection at B is Δ, when the load at B is removed. When the load at A is removed, the deflection at A will be

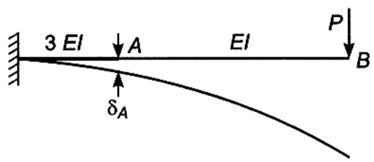


(A) $\frac{\Delta}{4}$ (B) $\frac{\Delta}{2}$ (C) $\frac{2\Delta}{3}$ (D) Δ

Soln. (D)
Deflection at B, when load at B is removed



Deflection at A, when load at A is removed



From Maxwell's reciprocal theorem,
$\delta_A = \Delta$, as portion AB has constant EI.

SLOPE DEFLECTION METHOD

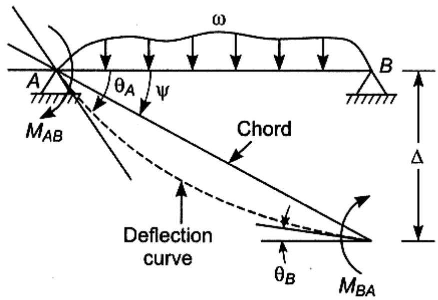
It is a displacement method of analysis. Equations are written in terms of the unknown displacements. By forming slope deflection equations and applying joint and shear equilibrium conditions, the **rotation angles** (or the slope angles) are calculated.

Slope Deflection Equations

The foregoing equilibrium equations can be expressed in terms of the unknown joint rotations, Q_B and Q_C by using slope-deflection equations that relate member and moments to the unknown joint rotations.

Sign Conventions of Slope Deflection Equations

- Clockwise moments and angular displacement (θ) will be considered as **positive**.
- Linear displacement (Δ) is considered positive when span's chord angle (ψ) is clockwise.



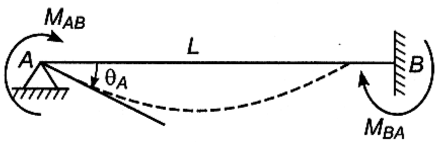
Span's chord angle diagram

Determination of General Equation

Slope deflection can be obtained by using principle of superposition by considering **separately** the moments developed at **each support** due to

- angular displacement θ_A
- angular displacement θ_B
- linear displacement Δ
- external loads

Angular displacement at A, (θ_A)



External loads diagram

$M_{AB} = \dfrac{4EI}{L}\theta_A$;

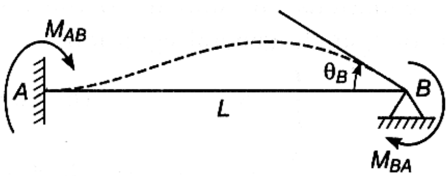
$M_{BA} = \dfrac{2EI}{L}\theta_A$ (clockwise, so positive)

where, EI = flexural rigidity,

M_{AB} = Moment about AB (counter clockwise)

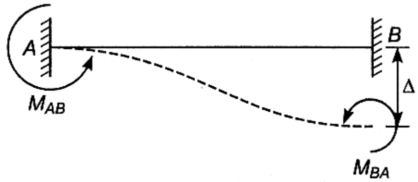
Angular displacement at (B, θ_B)

$M_{AB} = \dfrac{2EI}{L}\theta_B$, $M_{BA} = \dfrac{4EI}{L}\theta_B$ (clockwise, so + ve)



External loads diagram

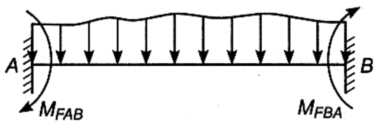
Linear displacement at B, (Δ)
Ends A and B do not rotate, only there is relative linear displacement of end B w.r.t., A.



Movable moment diagram

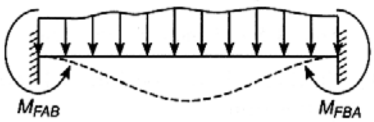
$M_{AB} = \frac{-6EI\Delta}{L^2}$, $M_{BA} = \frac{-6EI\Delta}{L^2}$ (anti-clockwise moments, so negative)

Fixed end moments due to applied loads
Let us choose any arbitrary loading on AB.



Fixed end moments

Here we have taken M_{FAB} and M_{FBA} both clockwise only for convenience in solving problems.
Actually,



Arbitrary loads diagram

Combining all four (a), (b), (c), (d)

At end A Final moment,

$$M_{AB} = \frac{4EI}{L}\theta_A + \frac{2EI}{L}\theta_B - \frac{6EI}{L^2}\Delta + M_{FAB}$$

or $$M_{AB} = 2E\left(\frac{I}{L}\right)\left[2\theta_A + \theta_B - 3\left(\frac{\Delta}{L}\right)\right] + M_{FAB}$$

At end B Final moment,

$$M_{BA} = \frac{2EI}{L}\theta_A + \frac{4EI}{L}\theta_B - \frac{6EI\Delta}{L^2} + M_{FAB}$$

or $$M_{BA} = 2E\left(\frac{I}{L}\right)\left[\theta_A + 2\theta_B - 3\left(\frac{\Delta}{L}\right)\right] + M_{FAB}$$

Since, these equations are similar, so the result can be expressed in one single general equation.

Let, $\frac{I}{L} = K$ (member stiffness)

$\frac{\Delta}{L} = \psi$ (span's chord rotation)

Referring one end as near end (N) and other as far end (F)
General slope deflection equation,

$$M_N = 2\,EK(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

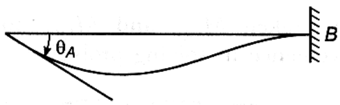
where, θ_N = Angular displacement at near end
θ_F = Angular displacement at far end
M_N = Moment about near end

Determination of Rotation at Any End Support

Fixed end has a property that no slope is generated. Hence, moment are applied in the sense, so as to keep member at original position.

Original moment diagram

We can visualize this as if there is no fixed support at A then θ_A is generated at A.



Fixed moment diagram

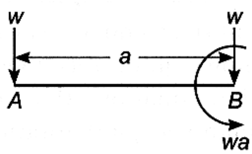
So, keep $\theta_A = 0$, there should be internal moment at A opposite to θ_A direction.

Example 24. What is the value of θ_B for the beam shown in figure?

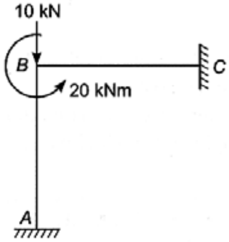
(A) zero
(B) $\frac{15}{EI}$ anti-clockwise
(C) $\frac{30}{EI}$ anti-clockwise
(D) $\frac{30}{EI}$ anti-clockwise

Soln. (B)

Force transmission concept A force can be transmitted to new position by keeping its magnitude and direction same as follows.



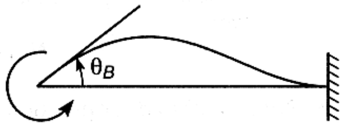
A → original location; B → new shifted position



Slope deflection equation for beam span BC

$$M_{FBC} = 0$$

$\theta_C = 0$
$\psi = 0$ as $\Delta = 0$



$M_{BC} = (FEM)_{BC} + 2\ EK(2\theta_N + \theta_F - 3\psi)$

$M_{BC} = 0 + 2\ E\left(\frac{I}{6}\right)[2\ \theta_B + 0 - 0]$

i.e., $M_{BC} = \frac{4\,EI}{6}\theta_B$

Slope deflection equation for column AB

$(FEM)_{BA} = 0,\ \theta_A = 0,\ \psi = 0$

$M_{BA} = (FEM)_{BA} + 2\ EK(2\ \theta_N + \theta_F - \psi)$

$= 0 + 2E\left(\frac{I}{6}\right)(2\theta_B + 0 - 0)$

or $M_{BA} = \frac{4\,EI}{6}\theta_B$

For equilibrium of joint B

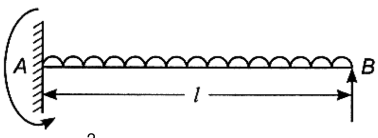
$M_{BA} + M_{BC} = 20$ kN-m (anti-clockwise)

or $\frac{4\,EI}{6}\theta_B + \frac{4\,EI}{6}\theta_B = 20$

$\left\{\theta_B = \frac{15}{EI}\right\}$ anti-clockwise

Suffix N → near end i.e., B
F → far end i.e., A and C

Example 25. The propped cantilever AB carries a udL of q/length. In this condition the moment reaction at A, $M_A = \frac{q\ell^2}{8}$. What is the clockwise moment required at B to make the slope of the deflection curve equal to zero?



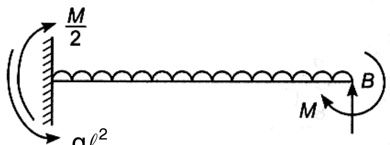
$M_A = \frac{q\ell^2}{8}$

(A) $\frac{q\ell^2}{8}$ (B) $\frac{q\ell^2}{6}$ (C) $\frac{q\ell^2}{12}$ (D) $\frac{q\ell^2}{4}$

Soln. (C)

Method 1

Let the clockwise moment required to make the slope of the deflection curve equal to zero to be M. Thus a carry over moment of magnitude M/2 will be induced A, in clockwise direction.



$M_A = \frac{q\ell^2}{8}$

Taking moment about B = 0

$$\frac{-q\ell^2}{8} + \frac{M}{2} + M = 0$$

or $M = \frac{q\ell^2}{12}$ (clockwise moments as positive)

Method 2

Slope deflection equation at B

$$M_{BA} = (FEM)_{BA} + \frac{2E\ell}{L}(2\theta_B + \theta_A - \psi)$$

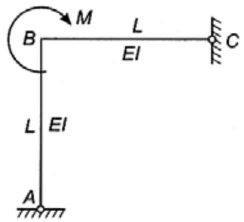
or $$\left[M_{BA} = \frac{q\ell^2}{12} + \frac{4E\ell}{L}\cdot\theta_B\right]$$

Our aim is to make $\theta_B = 0$

putting $\theta_B = 0$ in above equation

$M_{BA} = \frac{q\ell^2}{12}$; if we apply this amount of clockwise moment at B. Slope θ_B becomes zero.

Example 26. What is the rotation of the member at C for a frame as shown in figure below?

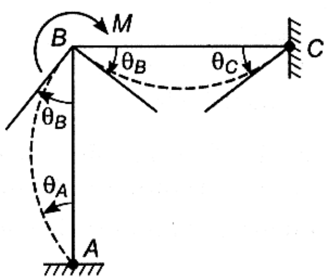


(A) $\frac{ML}{3EI}$ (B) $\frac{ML}{4EI}$ (C) $\frac{ML}{6EI}$ (D) $\frac{ML}{12EI}$

Soln. (D)

Ends A and C are hinged.

So, final moments



i.e., $M_{CB} = 0$, $M_{AB} = 0$

Slope deflection equation for span BC

$$M_{BC} = (FEM)_{BC} + \frac{2EI}{L}(2\theta_B + \theta_C)$$

$$M_{BC} = 0 + \frac{2EI}{L}(2\theta_B + \theta_C) \quad ....(i)$$

$$M_{CB} = (FEM)_{CB} + \frac{2EI}{L}(2\theta_C + \theta_B)$$

$$0 = 0 + \frac{2EI}{L}(2\theta_C + \theta_B)$$

$$\Rightarrow \quad \theta_C = \frac{-\theta_B}{2} \quad ....(ii)$$

Slope deflection equation for column AB

$$M_{BA} = (FEM)_{BA} + \frac{2EI}{L}(2\theta_B + \theta_A)$$

$$M_{BA} = 0 + \frac{2EI}{L}(2\theta_B + \theta_A) \quad ....(iii)$$

or $$M_{AB} = (FEB)_{AB} + \frac{2EI}{L}(2\theta_A + \theta_B)$$

$$0 = 0 + \frac{2EI}{L}(2\theta_A + \theta_B)$$

$$\Rightarrow \quad \theta_A = \frac{-\theta_B}{2} \quad ....(iv)$$

Equilibrium equation at joint B

$$M_{BA} + M_{BC} = M$$

or $$\frac{2EI}{L}(2\theta_B + \theta_A) + \frac{2EI}{L}(2\theta_B + \theta_C) = M \quad ....(v)$$

Replacing values of θ_A = and θ_C from Eqs. (ii) and (iv) in Eq. (v)

$$\frac{2EI}{L}\left(2\theta_B - \frac{\theta_B}{2}\right) + \frac{2EI}{L}\left(2\theta_B - \frac{\theta_B}{2}\right) = M$$

or $$\theta_B = \frac{ML}{6EI}$$

Therefore, $\theta_C = \frac{-\theta_B}{2} = \frac{-ML}{12EI}$ (anti-clockwise)

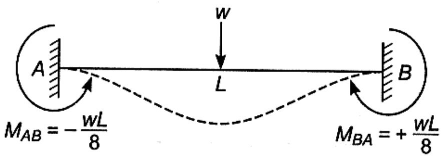
(FEM) are zero as there is no span loading.

MOMENT DISTRIBUTION THEOREM

In the moment distribution method, every joint of structure to be analysed is fixed so as to develop the fixed-end moments. Then, each fixed joint is sequentially released and the fixed-end moments (which by the time of release are not in equilibrium) are distributed to adjacent members until equilibrium is achieved. The moment distribution method in mathematical terms can be demonstrated as the process of solving a set of simultaneous equations by means of iteration. The moment distribution method falls into the category of displacement method of structural analysis.

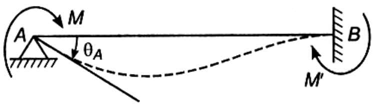
Sign Conventions of Moment Distribution Theorem
Clockwise moments acting on the member are **positive**.

Fixed End Moments (FEMs)
The moments at the walls or fixed joints of a loaded member are called **fixed end moments**.
e.g.,



Fixed end moments

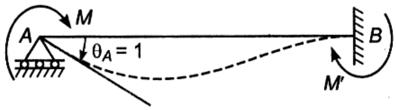
Member Stiffness Factor
Consider the beam as shown in figure, which is pinned at one end and fixed at the other. Application of the moment M causes the end A to rotate through an angle θ_A,



Moveable moment diagram

$$M = \frac{4EI}{L}.\theta_A$$

Absolute Stiffness of Member [K]
The moment M required to produce unit slope ($\theta_A = 1$) is called stiffness of member AB.
For end fixed



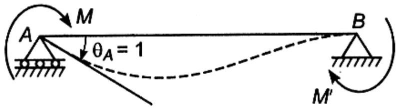
Fixed end diagram

e.g., $M_{AB} = \frac{4EI}{L}.\theta_A$

For $\theta_A = 1$; $K = \frac{4EI}{L}$

For end hinged

e.g.,



Hinged end diagram

$$M_{AB} = \frac{3EI}{L}\theta_A$$

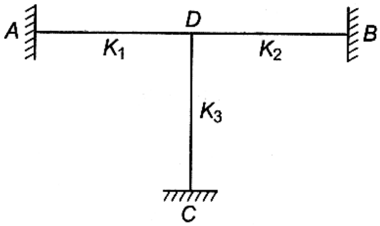
For $\theta_A = 1;\ K = \frac{3EI}{L}$

Joints Stiffness Factor

If several members are fixed connected at a joint and each of their far ends is fixed, then by principle of super position, the total stiffness factor at the joint is the sum of the member stiffness factors at the joint.

i.e., $K_T = \Sigma K$

e.g.,



Joints ends diagram

Total stiffness at joint D

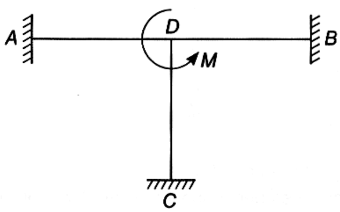
$$K_T = K_1 + K_2 + K_3$$

where, K_1, K_2 and K_3 are the stiffness factor in AD, DB, DC frames respectively.

Distribution Factor [DF]

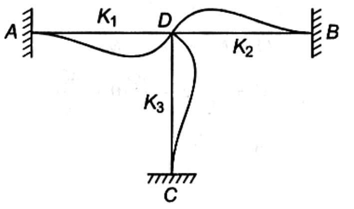
If a moment M is applied to a fixed connected joint, the connecting members will each supply a portion of the resisting moment necessary to satisfy moment equilibrium at the joint. That fraction of the total resisting moment supplied by the member is called **distribution factor** (DF).

e.g.,



Fixed ends diagram

Deflected shape of above diagram is



Deflected shape

If a moment M is applied at fixed joint C, this causes the joint to rotate an amount θ, then each member connected to that joint C, rotates by this same amount θ.
For member AD : $M_1 = K_1\theta$
For member BD : $M_2 = K_2\theta$
For member CD : $M_3 = K_3\theta$
for equilibrium condition at C

$$M = M_1 + M_2 + M_3$$
$$= \theta\Sigma K$$

Then, the distribution factor for the member i th member is

$$DF_i = \frac{M_i}{M}$$
$$= \frac{K_i\theta}{\theta\Sigma K_i}$$

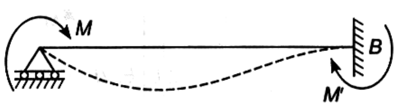
or $$DF = \frac{K_i}{\Sigma K_i}$$

Member Relative Stiffness Factor
In general, modulus of elasticity E will be the same for all the members. If this is the case, the common factor 4E in equation $\left(M = \frac{4EI}{L}\right)$ will **cancel** from the **numerator** and **denominator** of equation $\left(DF = \frac{K_i}{\Sigma K_i}\right)$.
Hence, member's relative stiffness factor

$$K_R = \frac{I}{L} \text{ for end fixed}$$
$$K_R = \frac{3I}{4L} \text{ for end hinged}$$

Use K_R values in distribution factor calculations.

Carry-Over Factor [CO]
When a moment M is applied to produce rotation without translation at the near supported end A of a beam whose further end B is fixed.



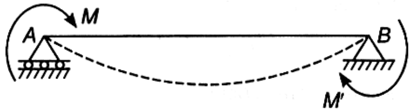
Carry-over diagram

Carry over moment at end fixed end B

$$M' = \frac{M}{2}$$

It has the same sense (direction) as the applied moment Carry over factor = $\frac{+1}{2}$

If the farther end B is hinged.



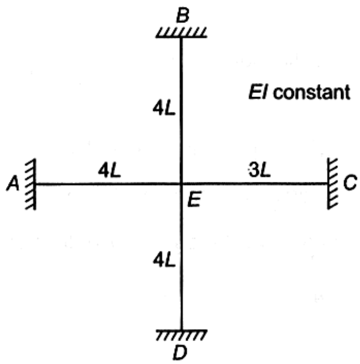
Hinged end

Carry over moment at end B

$$M' = 0$$

∴ Carry over factor = 0

Example 27. Distribution factor for BE in given figure is



(A) $\frac{1}{4}$ (B) $\frac{3}{13}$ (C) 1.24 (D) 0.4

Soln. (A)

Distribution factor for any i the member

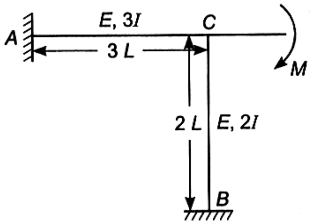
$$DF = \frac{K_i}{\Sigma K_i}$$

$$\Sigma K_i = K_{AE} + K_{BE} + K_{CE} + K_{DE}$$

$$= \left(\frac{I}{4L}\right) + \left(\frac{I}{4L}\right) + \frac{3}{4}\left(\frac{I}{3L}\right) + \left(\frac{I}{4L}\right) = \frac{I}{L}$$

Distribution factor for member BE = $\frac{\left(\frac{I}{4L}\right)}{\frac{I}{L}} = \frac{1}{4}$

Example 28. Moment A and B in the following figure will satisfy which of the following condition.



(A) $|M_A| = |M_B|$ (B) $|M_A| > |M_B|$

(C) $|M_A| = \frac{M}{4}$ (D) $|M_A| = |M_B| = \frac{M}{4}$

Soln. (D)

Relative stiffness of member AC

$$K_{R_1} = \frac{3I}{3L} = \frac{I}{L}$$

Relative stiffness of member BC

$$K_{R_2} = \frac{2I}{2L} = \frac{I}{L}$$

Since, relative stiffness of both member moment in both members AC and BC will be equally distributed and the half of moment at joint C.

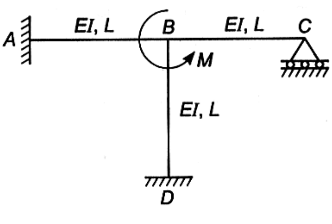
As, distribution factor $DF_{AC} = \frac{K_{R_1}}{\Sigma K} = \frac{\frac{I}{L}}{\frac{I}{L}+\frac{I}{L}} = \frac{1}{2}$

Similarly, Distribution factor $DF_{BC} = \frac{K_{R_2}}{\Sigma K} = \frac{\frac{I}{L}}{\frac{I}{L}+\frac{I}{L}} = \frac{1}{2}$

Now, moment M is applied at force end far from joint C as joint C is fixed (rigid), so carry over moment at joint C = $\frac{M}{2}$

Half of moment at C will be carried over to fixed ends A and B i.e., $|M_A| = |M_B| = \frac{M}{4}$

Example 29. All members of the frame shown below have the same flexural rigidity EI and length L. If a moment M is applied at joint B, the rotation of the joint is



(A) $\frac{ML}{12EI}$ (B) $\frac{ML}{11EI}$ (C) $\frac{ML}{8EI}$ (D) $\frac{ML}{7EI}$

Soln. (B)

Stiffness of AB $K_1 = \frac{4EI}{L}$

Stiffness of CB $K_2 = \frac{3EI}{L}$

Stiffness of DB $K_3 = \frac{4EI}{L}$

Since, all joint B is rigid (fixed).

So all the member connected at B will be rotated by same angle θ.

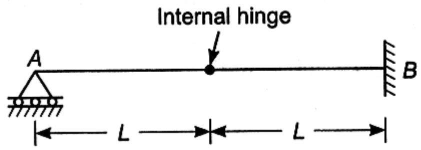
$$M_1 = K_1\theta$$
$$M_2 = K_2\theta$$
$$M_3 = K_3\theta$$

or $$M = M_1 + M_2 + M_3$$

$$M = \frac{4EI}{L}\theta + \frac{3EI}{L}\theta + \frac{4EI}{L}\theta$$

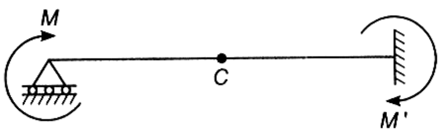
$$\theta = \frac{ML}{11EI}$$

Example 30. Carry over factor C_{AB} for the beam shown in the figure below is

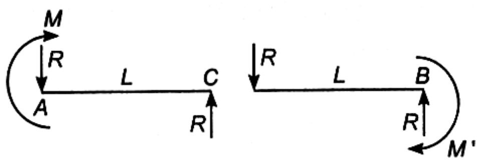


(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

Soln. (D)



Let a clockwise moment M is applied at end A and M′ be carry over moment at B.



From FBD of AC

$$\frac{M}{L} = R$$

From FBD of CB

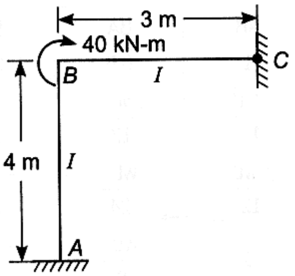
$$RL = M'$$

$\Rightarrow \quad M' = \frac{M}{L}.L$

$M' = M$

Therefore, carry over factor = 1

Example 31. For the rigid frame shown below, what is the moment reaction at A?



(A) 5 kN-m (B) 10 kN-m (C) 12.33 kN-m (D) 15 kN-m

Soln. (B)

Relative stiffness of member BC

$K_1 = \frac{3}{4}\left(\frac{I}{3}\right)$ as end C is hinged

Relative stiffness of member BA

$K_2 = \frac{I}{4}$

$\Sigma K = K_1 + K_2$

$= \frac{I}{4} + \frac{I}{4} = \frac{I}{2}$

Distribution factor (DF) for member BC

$$(DF)_{BC} = \frac{\frac{I}{4}}{\frac{I}{2}} = \frac{1}{2}$$

Similarly, $(DF)_{BA} = \frac{\frac{I}{4}}{\frac{I}{2}} = \frac{1}{2}$

i.e., $M_{BA} = (DF)_{BA} \times 40$ kN-m

$= 20$ kN-m

Thus by carry over theorem

Moment at A $= \frac{1}{2} M_{BA}$

$= \frac{20}{2}$

$= 10$ kN-m

Moment Distribution for Beams

Like the slope deflection method, the moment distribution method can be used only for the analysis of continuous beams and frames, taking into account their bending deformations only.

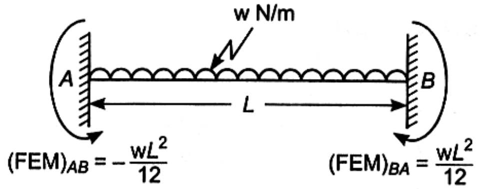
Objective To find out the end moments on beam spans.

Identity The joints on the beam

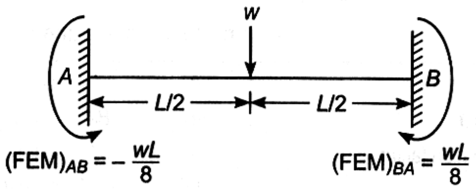
Calculate The stiffness factor K and distribution factors for each span at the identified joint. Also determine the Fixed End Moments (FEM), for each loaded span. Since clockwise direction is assumed + ve.

e.g.,

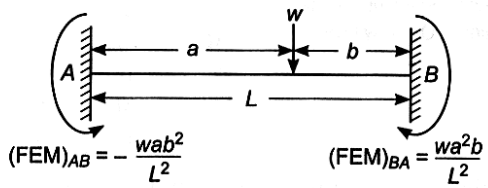
(a)



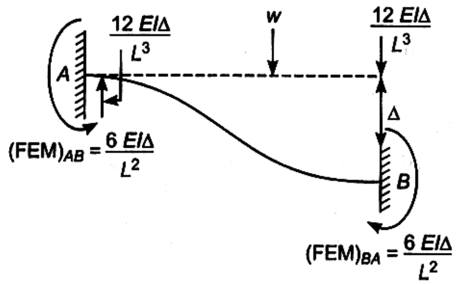
(b)



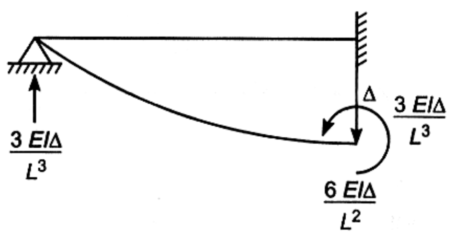
(c)



(d)



(e)

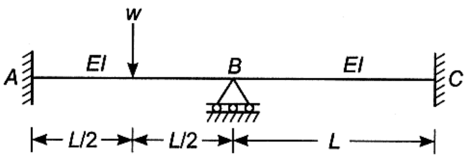


Steps to Solve the Problems under Moment Distribution for Beams

The following steps are involved to solve problem under moment distribution for beams.

- Assume all identified joints are initially locked.
- Find out the unbalanced moment that is needed to put each joint in locked position.
- Now apply an equal but opposite unbalanced moment to unlock the joint.
- Distribute this unbalancing moment into the connecting spans to the joint.
- Carry these moments in each span over to its other end by multiplying carry over factor $\left(+\frac{1}{2}\right)$.
- Finally, by repeating this cycle of locking and unlocking of joints, it will be found that moment corrections will diminish, since the beam tends to achieve its final deflected shape.
- All calculations are performed in tabulated form.
  For objective level problems only one cycle is sufficient.

Standard Table



A simple load diagram

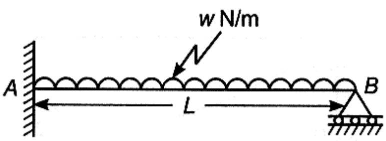
Joint	A	B		C
Member	AB	BA	BC	CB
Distribution factor	0	0.5	0.5	0
FEM (Finite Element Method) distribution, CO (Carry Over Factor)	$\frac{-wL}{8}$ $\frac{-wL}{32}$	$\frac{+wL}{8}$ $\frac{-wL}{16}$	$\frac{-wL}{16}$	$\frac{-wL}{32}$
ΣM	$\frac{-5wL}{32}$	$\frac{wL}{16}$	$\frac{-wL}{16}$	$\frac{-wL}{32}$

The identified joint should have finally equal and opposite moments from connecting spans for its equilibrium.

Example 32. A propped cantilever beam of span L is loaded with UDL of intensity w/unit length, all through span bending moment at the fixed end is

(A) $\frac{wL^2}{8}$ (B) $\frac{wL^2}{12}$ (C) $\frac{wL^2}{24}$ (D) $\frac{wL^2}{48}$

Soln. (A)

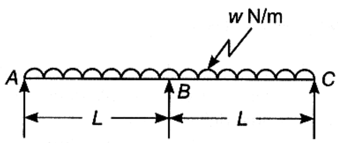


For pinned end, DF = 1

Joint	A	B
Member	AB	BA
FEM	$\frac{-wL^2}{12}$	$\frac{wL^2}{12}$
Distribution, CO	$\frac{-wL^2}{24}$	$\frac{-wL^2}{12}$
ΣM	$\frac{-wL^2}{8}$	0

Thus, $M_A = \frac{wL^2}{8}$ (anti-clock wise)

Example 33. For the beam shown below, the reaction at support B, from the span AB is

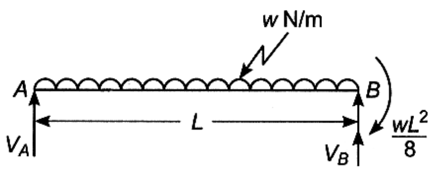


(A) $\frac{3wL}{8}$ (B) $\frac{5wL}{8}$ (C) $\frac{wL}{6}$ (D) $\frac{wL}{4}$

Soln. (B)
Using moment distribution table, finding out M_B

Joint	A	B		C
Member	AB	BA	BC	CB
Distribution factor	1	0.5	0.5	1
FEM	$\frac{-wL^2}{12}$	$\frac{wL^2}{12}$	$\frac{-wL^2}{12}$	$\frac{wL^2}{12}$
distribution, CO	$\frac{+wL^2}{12}$ →	$\frac{wL^2}{24}$	$\frac{-wL^2}{24}$	← $\frac{-wL^2}{12}$
ΣM	0	$\frac{wL^2}{8}$	$\frac{-wL^2}{8}$	0

FED of span AB



Moment of B is shown clockwise as it is positive for span BA (see table)
Taking moment of all forces about A = 0

$$V_B \times L - \frac{wL^2}{2} - \frac{wL^2}{8} = 0 \quad \text{or} \quad V_B = \frac{5wL}{8}$$

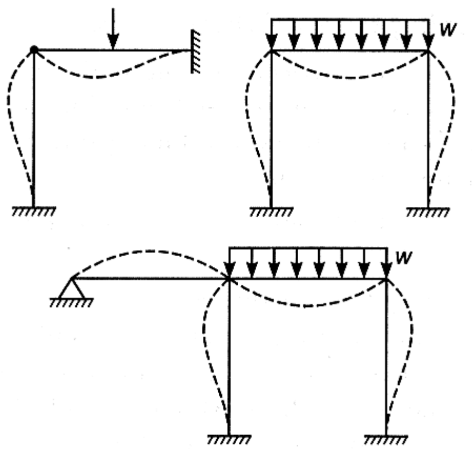
Moment Distribution for Frames

Framed structures using moment distribution method can be analysed as with sides way and without sidesway.
The both conditions for analysing framed structures are given below.

Condition 1
No Sidesway

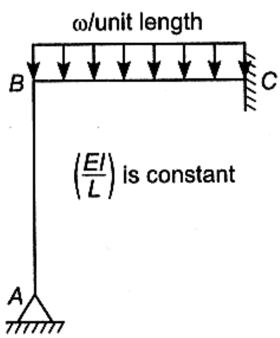
Adopt the same procedure, as for beams

- A frame will not sidesway (left or right displacement), provided it is properly restrained.
- If it is unrestrained, then no sidesway will occur, provided it is symmetric with respect to loading and geometry both.



Moment distribution frame

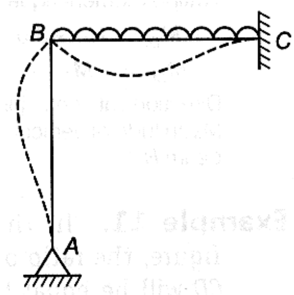
Example 34. What is the ratio of magnitudes of moments in the member BC at the ends B and C in the figure given below?



(A) 1 : 1 (B) 3 : 1 (C) 3 : 4 (D) 1 : 3

Soln. (D)

Identified joint → B



Stiffness of member BA : $K_1 = 3\left(\frac{EI}{L}\right)$

Stiffness of member BC : $K_2 = \frac{4EI}{L}$

Distribution factor for member BA

$$= \frac{K_1}{K_1 + K_2} = \frac{3}{7}$$

Distribution factor for member BC

$$= \frac{K_2}{K_1 + K_2} = \frac{4}{7}$$

Moment Distribution Table

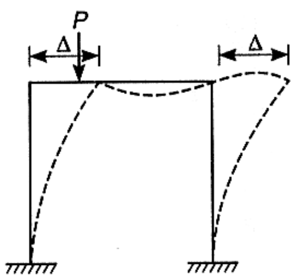
Joint	A	B		C
Member	AB	BA	BC	CB
Distribution factor	1	$\frac{3}{7}$	$\frac{4}{7}$	0
FEM	0	0	$\frac{-wL^2}{12}$	$\frac{wL^2}{12}$
distribution factor, CO	0	$\frac{wL^2}{28}$	$\frac{wL^2}{21}$	$\frac{wL^2}{42}$
ΣM	0	$\frac{wL^2}{28}$	$\frac{-wL^2}{28}$	$\frac{9wL^2}{84}$

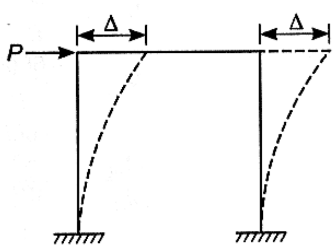
Thus, M_B : $M_C = \dfrac{\frac{wL^2}{28}}{\frac{9wL^2}{84}} = \frac{1}{3}$

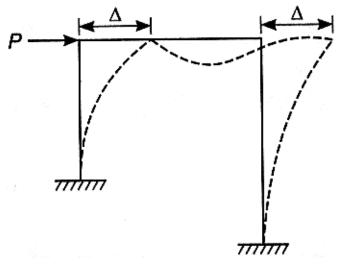
Condition

Sidesway

A frame will sidesway or be displaced to the side when it is non-symmetric or loading is non-symmetric.







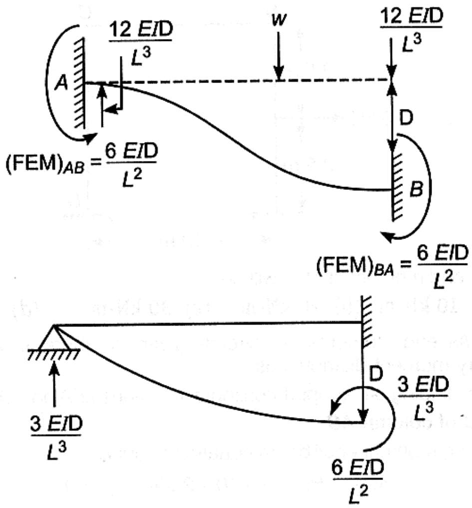
Moment distribution for frame in sideway

Causes of Sidesway

Following are the causes of sidesway

- Unsymmetrical loading
- Unsymmetrical outline
- Different end conditions
- Non-uniform section of members
- Horizontal loading on column
- Combination of all the above

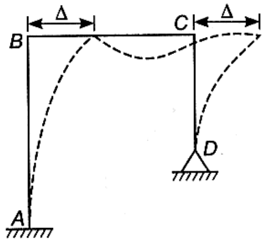
Example 35. The given figure shows a portal frame with one end fixed and other hinged. The ratio of the fixed end moments M_{BA}/M_{CD} due to sidesway will be



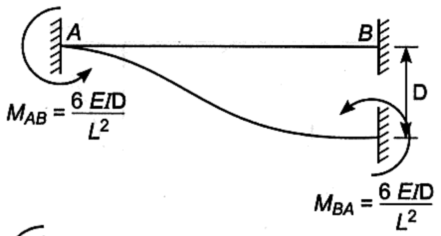
(A) 1.0 (B) 2.0 (C) 2.5 (D) 3.0

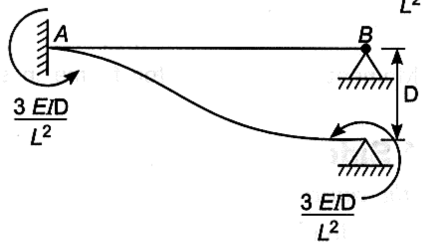
Soln. (A)

Due to S-way deflection of point B and point C will be equal.



As we know



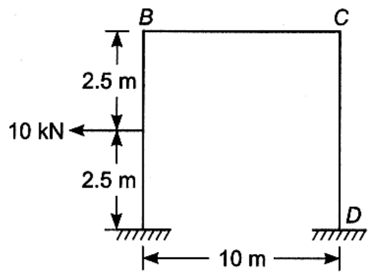


So, for frame

$$M_{BA} = \frac{-6(EI)\,\Delta}{L^2} \text{ and } M_{CD} = \frac{-3\left(\frac{EI}{2}\right).\,\Delta}{\left(\frac{L}{2}\right)^2} = \frac{-6\,EI\Delta}{L^2} \quad \{\because \text{ Anti-clockwise moment taken as} - \text{ve}\}$$

$$\therefore \quad \frac{M_{BA}}{M_{CD}} = 1.0$$

Example 36. For the portal frame shown in the figure below, the final end moments are M_{AB} = 15 kN-m, M_{BA} = 10 kN-m, M_{CD} = 20 kN-m

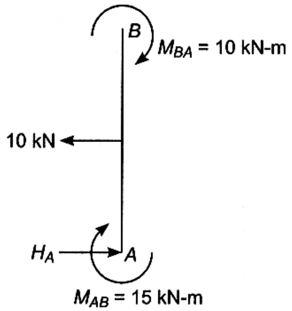


The end moment at M_{DC} will be
(A) 10 kN-m (B) 20 kN-m (C) 30 kN-m (D) 40 kN-m

Soln. (C)
As end moments are directly given, so there is no need to carry moment distributions.
Now, finding out support horizontal reactions at A and D.

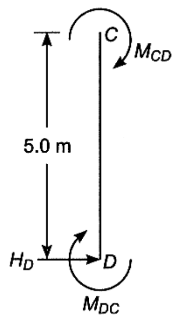
FBD of column AB
Writing moment equilibrium equation about B.

$$M_{AB} - H_A \times 5 - 10 \times 2.5 + M_{BA} = 0$$



or $$H_A = \frac{(M_{AB} + M_{BA}) - 10 \times 2.5}{5} = \frac{(15+10)-25}{5} = 0$$

FBD of column CD



Also $H_A + H_D = 10$ as $H_A = 0$

So, $H_D = 10$ kN

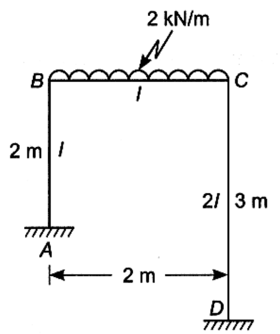
Writing moment equation about C

$$M_{DC} - H_D \times 5.0 + M_{CD} = 0$$

$$M_{DC} = -M_{CD} + 10 \times 5.0 = -20 + 50 = 30 \text{ kN-m}$$

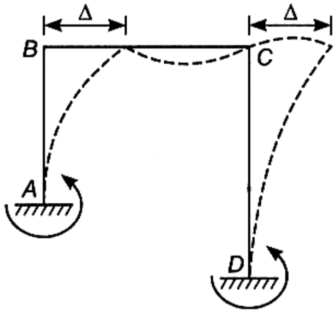
Direction at end joints are clockwise for positive moments. Magnitude of vertical reactions at A and D can be found by FBD of beam BC.

Example 37. In the portal frame shown in the given figure, the ratio of S-way moments in columns AB and CD will be equal to



(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{9}{8}$ (D) $\frac{13}{8}$

Soln. (C)
Loading is symmetrical but ends are not at same level, so this will cause sidesway of frame.



$$M_{AB} = \frac{-6EI\Delta}{L^2} = \frac{-6EI\Delta}{2^2}$$

$$M_{CD} = \frac{-6E(2I)\Delta}{L^2} = \frac{-12EI\Delta}{3^2}$$

$$\frac{M_{AB}}{M_{CD}} = \frac{\frac{-6EI\Delta}{2^2}}{\frac{-12EI\Delta}{3^2}} = \frac{9}{8}$$

ANALYSIS OF CABLES AND ARCHES

Cables carry applied loads and develop mostly tensile stresses. Cables near the end supporting experience bending moments and shear forces. Arches carry applied loads and develop mainly in plane compressive stresses.

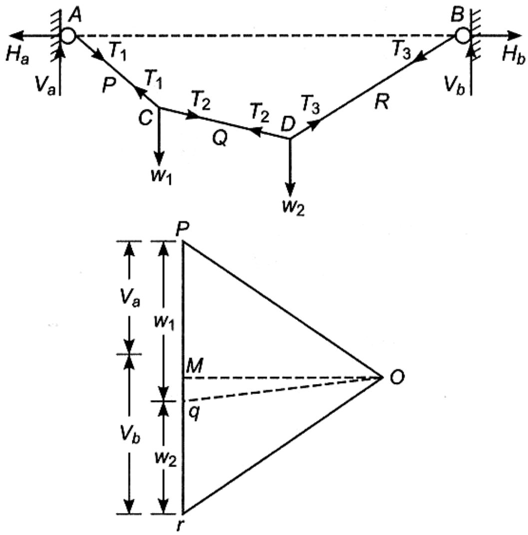
Cables
Cables are structural elements that resist loads by developing axial tensions. Cable is flexible and in-extensible, hence does not resist any bending moment or shear force (this is not always true e.g., fatigue of cables); self weight of cable neglected when external loads act on the cable.

Assumptions in Analysis of Cables

- Only axial tensile forces are carried by the cable, the force in the cable is tangential to the cable profile.
- Since it is in-extensible, the length is always constant; as a consequence of the cable profile not changing its length and form, it is assumed to be a rigid body during analysis.
- Even when a moving load is acting on the cable, the load is assumed to be uniformly distributed over the cable (since the cable profile is not assumed to change).
- During deriving basic relations between cable tension and cable slope. We assume that the cable is **perfectly flexible** and **inextensible**.
  i.e. Shear force and bending moment at every section of cable are zero. Thus, there is only cable tension is unknown internal force during analysis.

Funicular Polygon in Cables

The word funicular originates from the word funicle which means string. A string can resist only axial forces. This is a graphical method of finding the resultants of a system of forces, constructing BMDs, determining the rational shapes of arched and suspended systems and solving other problems of the statics of two-dimensional systems.



Funicular polygon

PQRO is a force polygon corresponding to the system of forces keeping the cable in equilibrium.

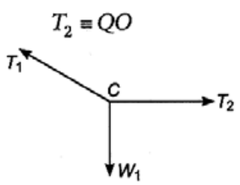
OM is line perpendicular to the load line PQR.

Let us consider equilibrium of joint C.

$T_1 \equiv$ Line PO

$w_1 \equiv$ Line PQ

Here T refers to tension cables., by principle of triangle



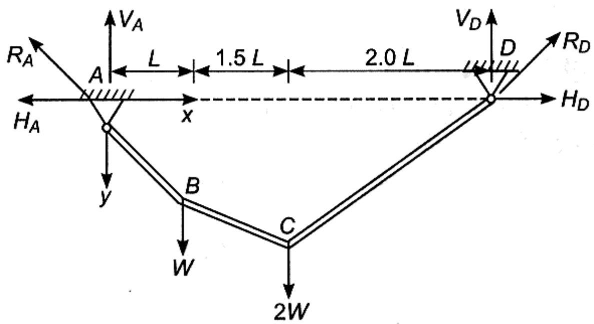
Equilibrium of joint C

It is easy to realise that horizontal components of each of T_1, T_2 and T_3 equals the horizontal reaction $H = H_a = H_b$.

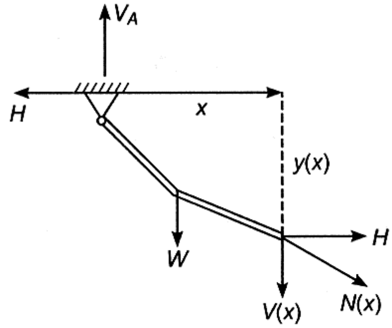
$H \equiv$ Line OM

Beam Analogy in Cables

The beam analogy is a conceptual tool for understanding how forces are distributed through a truss. The beam analogy works best with **parallel chord trusses** (horizontal top and bottom chords) but still provides insight for other types of trusses.



Cable



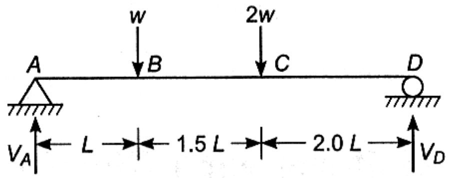
Cable free body diagram

Bending moment at any distance x from A.

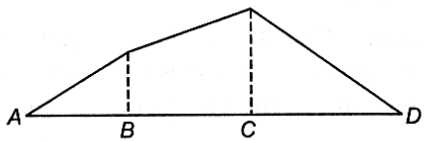
$$M(x) = \underbrace{V_A \,.\, x - w(x - L)}_{\text{beam moment}} - H \,.\, y(x)$$

$$M(x) = M_0(x) - H \,.\, y\,(x) \qquad \ldots \text{(i)}$$

or $\quad M(x) = \text{Beam moment} - \text{moment}$



Simple supported beam



BMD (beam moment)

As, we know that cable cannot resist any moment, thus from Eq. (i)

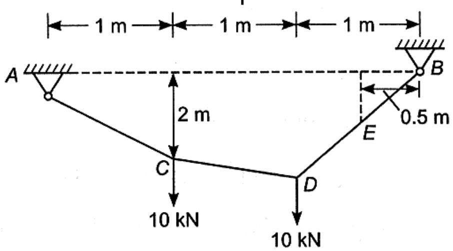
$$M(x) = 0$$

or $\quad M_0(x) - H \,.\, y(x) = 0$

or $\quad y(x) = \dfrac{M_0\,(x)}{H} \qquad \ldots \text{(ii)}$

as H is constant, profile cable at any location will have same shape as beam moment shape. If cable ordinates at any location are known, then horizontal reaction can be directly find out by Eq. (ii).

Example 38. The cable shown in the figure is loaded with loads 10 kN at C and 15 kN at D. What is ordinate of cable profile at section E of cable?

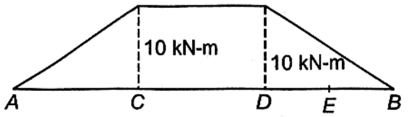


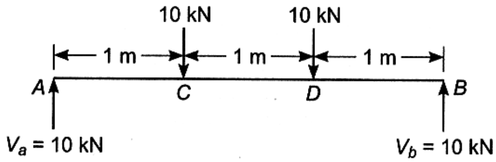
(A) 0.5 m (B) 1.0 m (C) 1.5 m (D) 2.0 m

Soln. (B)

Ordinate at section E = $\dfrac{\text{Beam moment at E}}{H}$

Beam





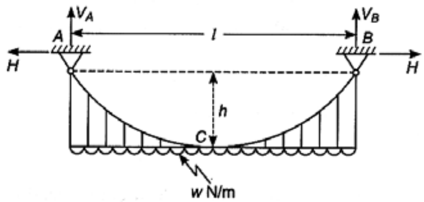
Beam moment

If ordinate at C is known, thus H = $\frac{\text{BM at C}}{2\text{m}} = \frac{10}{2} = 5\,\text{kN}$

Now, ordinate at section E = $\frac{\text{BM at E}}{\text{H}} = \frac{5\,\text{kNm}}{5\,\text{kN}} = 1\text{m}$

Cable Uniform with Distributed Loads (UDL)

UDL (Uniform Distributed Load) refers to the fact that the per square inch weight of the pallet or rack load does not vary from one point to another on the pallet.



Cable with UDL

Vertical Reaction $V_A = V_B = \frac{w\ell}{2}$

Horizontal Reaction

$$H = \frac{\text{Beam moment at C}}{\text{ordinate at C}} \left\{ \text{from } y(x) = \frac{M_0(x)}{H} \right\}$$

$$H = \frac{w\ell^2}{8h}$$

Where h = dip length of the cable

{since, BM at C = $\frac{w\ell^2}{8}$ for simply supported beam with udl w}

Maximum tension in cable, T_{max} = Resultant reaction at A or B

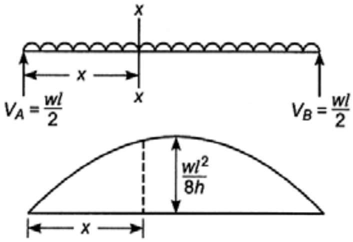
$$= \sqrt{V_A^2 + H^2}$$

$$= \sqrt{\left(\frac{w\ell^2}{2}\right) + \left(\frac{w\ell^2}{8h}\right)} = \frac{w\ell}{2}\sqrt{1 + \frac{\ell^2}{8h^2}}$$

Shape of Loaded Cable
As from previous analysis

Ordinate of profile y(x) = $\frac{\text{Beam moment}}{H}$

Simply supported beam



BMD of loaded cable

BMD
Beam moment at section x–x

$$= \frac{w\ell}{2}x - \frac{wx^2}{2}$$

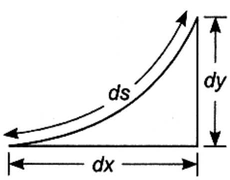
and horizontal reaction H = $\frac{w\ell^2}{8h}$

Then, $$y(x) = \frac{\frac{w\ell}{2}x - \frac{w\ell^2}{2}}{\frac{w\ell^2}{8h}}$$

$$y(x) = \frac{4hx}{\ell^2}[\ell - x]$$

This cable profile equation which is parabolic.

Length of Cable
The total length of cable (L) may be calculated by using the following considerations.
$(ds)^2 = (dx)^2 + (dy)^2$



or $$\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

where, ds = small curved length of the cable
If we take C as origin of parabola then, equation of cable profile

$$y = \frac{4hx^2}{\ell^2} \Rightarrow \frac{dy}{dx} = \frac{8hx}{\ell^2}$$

Thus, $\frac{ds}{dx} = \sqrt{1+\left(\frac{8hx}{\ell^2}\right)^2} = \left[1+\frac{64h^2x^2}{\ell^4}\right]^{1/2}$

$$\frac{ds}{dx} = 1 + \frac{1}{2}.\left(\frac{64h^2x^2}{\ell^4}\right)$$

or $\int_0^L ds = 2\int_0^{1/2}\left[1+\frac{32\,h^2x^2}{\ell^4}\right]dx$

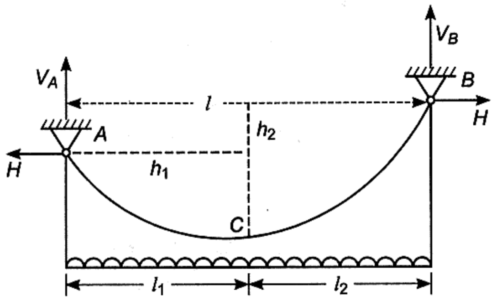
$\therefore$ Total length of the cables L = $\ell + \frac{8}{3}.\frac{h^2}{\ell}$

Ends of Cable at Different Level

Figure shows the total length of cable (L) has been braked into two i.e.,

$\ell_1 + \ell_2 = \ell$

Since, cable ACB is parabolic, thus we have



Total length of cable is braked

$$\frac{x^2}{y} = \text{constant}$$

or $\frac{\ell_1^2}{h_1} = \frac{\ell_2^2}{h_2}$

$\Rightarrow$ $\frac{\ell_1}{\sqrt{h_1}} = \frac{\ell_2}{\sqrt{h^2}} = \frac{\ell_1+\ell_2}{\sqrt{h_1}+\sqrt{h_2}}$

Horizontal reaction, H = $\frac{w\ell_2}{2\left(\sqrt{h_1}+\sqrt{h_2}\right)^2}$

where, h_1 and h_2 are the depth of the cables from supports A and B respectively.

Temperature Stress in the Cable

Length of cable L = $\ell + \frac{8h^2}{3\ell}$ … (i)

(from the above consideration)

where, ℓ is span and h is dip.

Let on increasing the temperature, the length of cable increases by dL, so that dip of the cable increases by dh.

Differentiating Eq. (i)

$$dL = \frac{16h}{3\ell}.dh$$

$$\therefore \quad dh = \frac{3\ell}{16h}dL$$

Let the rise of temperature be t°C

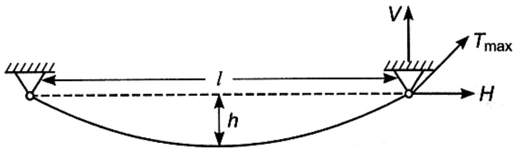
$$\therefore \quad dh = \frac{3\ell}{16h}\ell\,\alpha\,t$$

$$dh = \frac{3\ell^2}{16h}\alpha\,t$$

where α is a coefficient of thermal expansion in per °C.

Maximum tension in the cable $T_{max} = \sqrt{V^2 + H^2}$

When the central drip is small we can make approximation $T_{max} = H$



as, $\ell >> h$ cable is almost horizontal

Thus, $H >> V$ i.e., $T_{max} = H$

$$\text{Also } H = \frac{w\ell^2}{8h} \qquad \dots \text{(ii)}$$

as h increase, H decreases. When h increases to h + dh then H decreases to H − dH.

Differentiating Eq. (ii)

$$dH = \frac{-w\ell^2}{8h^2}.dh$$

$$\therefore \quad dH = \frac{-w\ell^2}{8h}.\frac{dh}{h}$$

$$dH = -H.\frac{dh}{h}$$

or $$\frac{dH}{H} = \frac{-dh}{h}$$

Stress in cable $f = \dfrac{T_{max}}{\text{Area (A)}} = \dfrac{H}{A}$

Let df be change in stress in the cable

$$\therefore \quad \frac{df}{f} = \frac{dH}{H} = \frac{-dh}{h} = \frac{-3\ell^2}{16h^2}.\alpha t$$

$$\therefore \quad \frac{df}{f} = \frac{-3\ell^2}{16h^2}.\alpha t$$

Example 39. A cable of span length 50 m has a dip of 5 m is subjected to a rise of temperature of 10°C. The cable supports of total load of 25 kN/m run of the horizontal span. What will be the change in tension due to rise of temperature?
(Take, $\alpha = 12 \times 10^{-6}$ per °C.)
(A) −7.03 kN (B) −3.51 kN (C) −2.82 kN (D) −1.91 kN
Soln. (B)

Horizontal reaction, $H = \dfrac{wL^2}{8h}$

$$= \frac{25 \times 50^2}{8 \times 5} = 1562.5 \text{ kN}$$

Change in horizontal reaction dH can be given as,

$$\frac{dH}{h} = \frac{-3}{16}\alpha\, t\, \frac{l^2}{h^2}$$

$$\therefore \quad dH = \frac{-3}{16}\alpha t \cdot \frac{l^2}{h^2} \cdot H$$

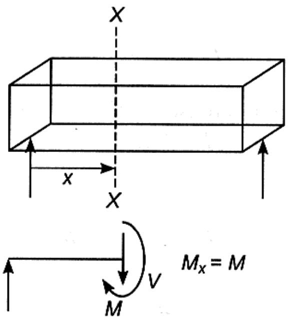
$$= \frac{-3}{16} \times \frac{12 \times 10^{-6} \times 10 \times 50^2 \times 1562.5}{5^2}$$

$$= -3.51 \text{ kN}$$

The change in horizontal reaction is practically the change in tension.

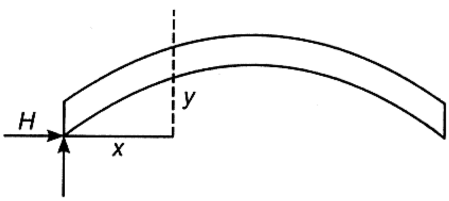
Arches

An arch is a structure that spans a space and supports structure and weight above it. A beam mainly resists the applied load by action of internal bending moments and shear force.



Rectangular bar showing as arches

Now, as we starts to provide an arching action, moment at any section starts reducing due to horizontal thrust moment at that section.



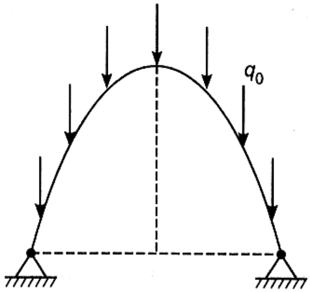
The arching action

$M_x = M - H.\, y$

Thus, arching effect reduces bending moment in the span. Hence, an arch is a structural member, which resists the applied load by action of normal thrust only.

Funicular Arch

Funicular arch is the inverse of the cable, supporting gravity loads by axial compression instead of axial tension. When an arch has parabolic shape and it is subjected to a uniform horizontally distributed load, then from analysis of cable it follows that only compressive forces will be resisted by the arch.

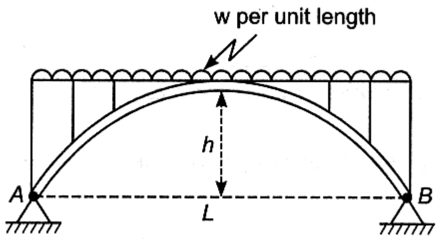


Ideal funicular shape

Under these conditions the arch shape is called the funicular arch, because no bending or shear forces occur in the arch.

Parabolic Arch

Free bending moment diagram in this case is parabola, having a maximum value $\frac{wL^2}{8}$ at the mid–span of equivalent simply supported beam. Thus, from cable concept



Parabolic arch

horizontal reaction,

$$H = \frac{\text{Beam moment}}{y(x)}$$

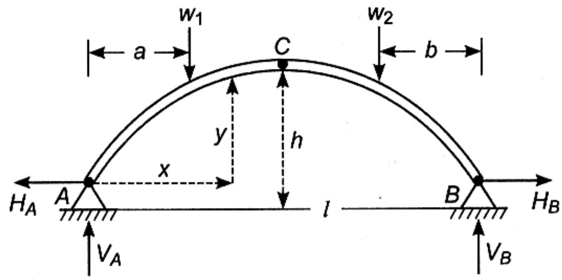
at C, $\quad H = \frac{wL^2}{8h}$

The equation of the parabolic arch

$y = \frac{4h}{\ell^2} \cdot x \cdot (\ell - x)$ where, x from end A.

Three Hinged Arches

If horizontal thrust is generated but the apex of the arch is a pin joint, this is termed a three–hinged arch. This type of a arch is used in making bridges type structures. A hinge is located at the crown. The arch shown in figure is subjected to a number of loads W_1, W_2. Let the reactions at A and B be (H, V_A) and (H, V_B) respectively. Since the BM at C is zero.



Three hinged arches forces and loads

Total unknowns forces developed, are V_A, H_A, V_B, H_B (i.e., four)

Available equations of equilibrium = $\sum F_x = 0$, $\sum F_y = 0$

$\sum M_B = 0$, $\sum M_C = 0$ (i.e. four)

Thus, all unknown forces can be find out.

$\sum F_y = 0$, $V_A + V_B = w_1 + w_2$ … (i)

$\sum F_x = 0$, $H_A = H_B$ … (ii)

$\sum M_A = 0$, $\sum M_B = 0$;

or $V_A . \ell - w_1 (\ell - a) - w_2 . b = 0$ … (iii)

$\sum M_C = 0$ ((hinge), from left side)

$$V_A . \frac{\ell}{2} + H_A . h - w_1\left(\frac{\ell}{2} - a\right) = 0 \quad \text{… (iv)}$$

From right side $V_B . \frac{\ell}{2} + H_B . h - w_2\left(\frac{\ell}{2} - b\right) = 0$ … (v)

On solving all these equations, we can get all support unknowns.

Let us consider portion AC of arch and writing the bending moment at any section X of arch

$M_x = V_a x - w_1 (x - a) - H. y$

$M_x = V_a . x - w_1 (x - a) - H . y$

Beam Moment

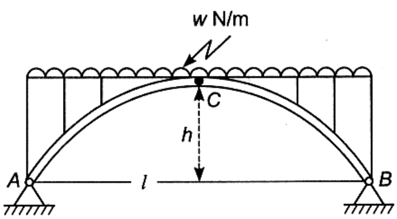
Actual bending moment at X = Beam moment at X – H moment at X

Three Hinge Parabolic Arch

Loaded with udl

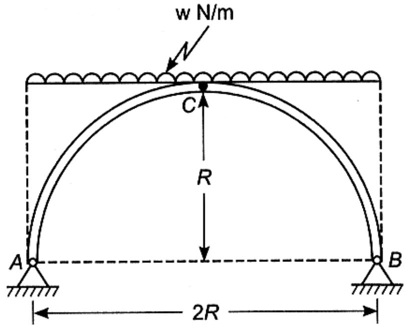
Actual bending moment at C = Beam moment at C – H moment at as C is hinge

$$\therefore \quad 0 = \frac{w\ell^2}{8} - H.h \text{ or } H = \frac{w\ell^2}{8h}$$



Parabolic arch loaded with udl

Three Hinged Semicircular Arch Loaded with udl



Three hinged semicircular arch under udl

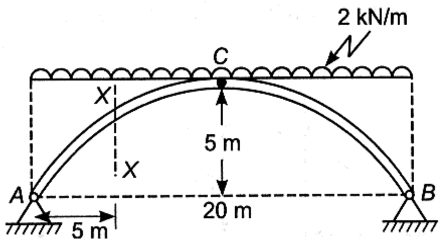
Actual bending moment at C = Beam moment at C − H moment at C = Beam moment at C − H moment at C (C is hinged) 0 = $\frac{w(2R)^2}{2}$ = H . R or H = $\frac{wR}{2}$

Here, R is a dip length.

Example 40. A three hinged symmetrical parabolic arch of span 20 m and rise 5 m carries a uniformly distributed load of 2 kN/m for the whole span. The bending moment at quarter point is

(A) 75 kN–m (hogging)
(B) 75 kN–m (sagging)
(C) 100 kN–m (sagging)
(D) zero

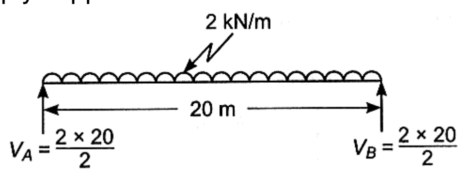
Soln. (D)



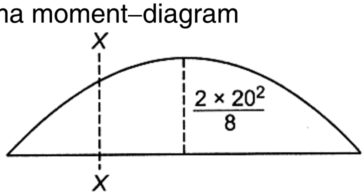
Bending moment at quarter point i.e., at section X = ?

Now, actual bending moment at X = Beam moment at X − H moment at X

Simply supported beam



Bema moment–diagram



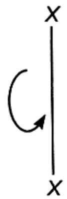
Beam moment at X = $V_A \times 5 - \dfrac{w \times 5^2}{2}$

$$= 20 \times 5 - \frac{2 \times 5^2}{2} = 75 \text{ kN–m (sagging)}$$

H moment at X = H . y

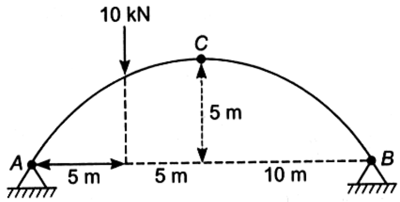
where, $H = \dfrac{wL^2}{8h} = \dfrac{2 \times 20^2}{8 \times 5} = 20$ kN

$$y = \frac{4hx}{l_2}(l - x) = \frac{4 \times 5 \times 5}{20^2}(20 - 5) = 3.75 \text{ m}$$



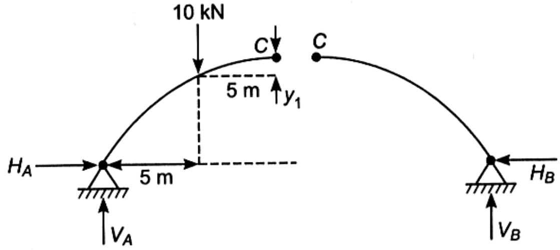
Thus, H moment at X = 20 × 3.75 = 75 kN–m (hogging)
Hence, Actual bending moment at X = 75 − 75 = 0

Example 41. A three hinged parabolic arch having a span of 20 m and a rise of 5 m carries a point load of 10 kN at quarter span from the left end as shown in the figure. The resultant reaction at the left support and it inclinations with the horizontal are respectively.



(A) 9.01 kN and 56.31°
(B) 9.01 kN and 33.69°
(C) 7.50 kN and 56.31°
(D) 2.50 kN and 33.69°

Soln. (A)



$V_A + V_B = 10$ … (i)

$H_A = H_B = H$ … (ii)

From left side (arch AC) $\sum M_C = 0$

$V_A \times 10 - H_A \times 5 - 10 \times 5 = 0$ … (iii)

or $10\,V_A - 5H = 50$ … (iv)

From right side of hinge C

$\sum M_C = 0$

$V_B \times 10 = 5 \times H$ … (v)

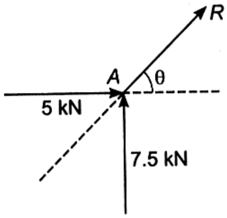
On solving Eq. (i), (iii) and (iv)

We get, $V_A = 7.5$ kN and $V_B = 2.5$ kN

$H = 5$ kN

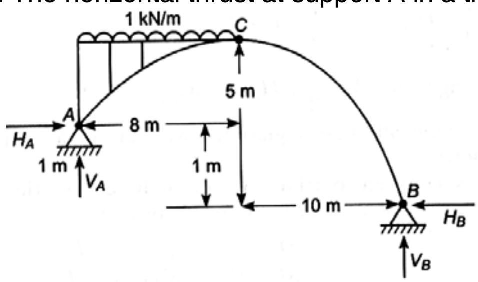
Resultant reaction at A

$R = \sqrt{5^2 + 7.5^2} = 9.01$ kN



$\tan\theta = \frac{7.5}{5}$ or $\theta = 56.31°$

Example 42. The horizontal thrust at support A in a three hinged arch shown in figure is



(A) 2 kN (B) 4 kN (C) 8 kN (D) 10 kN

Soln. (B)

$V_A + V_B = 1 \times 8$ … (i)

$H_A \neq H_B$ (as supports are not same level)

$\sum M_C = 0$ (from left side)

$V_A \times 8 - H_A \times 4 - \frac{1 \times 8^2}{2} = 0$ … (ii)

$\sum M_C = 0$ (from right side)

$V_B \times 10 - H_B \times 5 = 0$

Taking moment of all forces about A = 0

$V_B \times 18 - H_B \times 1 - 1 \times \frac{8^2}{2} = 0$ … (iii)

On solving all equations

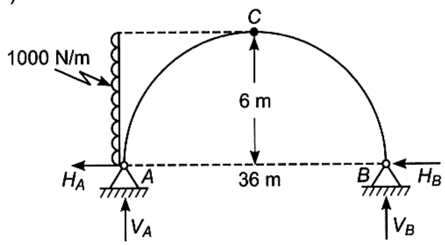
We get, $V_B = 2$ kN, $V_A = 6$ kN,

$H_B = 4$ kN, $H_A = 4$ kN

Example 43. A circular segmental three–hinged arch of span 36 m and a rise of 6 m is hinged at the crown. It carries a horizontal load 1000 N/m covering full height of the arch on the left side. The horizontal thrust at the right support will be

(A) 6000 N (B) 4500 N (C) 3000 N (D) 1500 N

Soln. (D)



$V_A + V_B = 0$ … (i)

$\sum M_C = 0$ (from left side of C)

$$18V_A + 6H_A = \frac{1000 \times 6^2}{2}$$

or $3V_A + H_A = 3000$ ... (ii)

$\sum M_C = 0$ (from right side of C)

$18V_B = 6H_B$

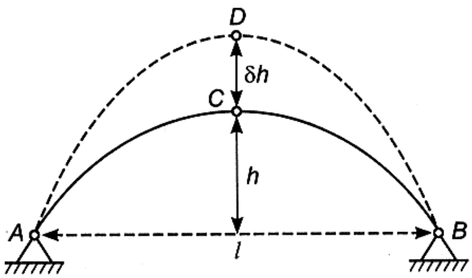
or $H_B = 3V_B$ … (iii)

$H_A + H_B = 1000 \times 6$ … (iv)

On solving equations $H_A = 4500$ N and $H_B = 1500$ N

Temperature Effect on
Three Hinged Arch

The rise of temperature increases the length of the arch. Since, the ends A and B do not move and hinge C is not connected to any permanent object, the crown will rise from C to D i.e., (C point is not fixed due to the variation in length which is just because of rise of temperature)



Effect of increasing in temperature on arch

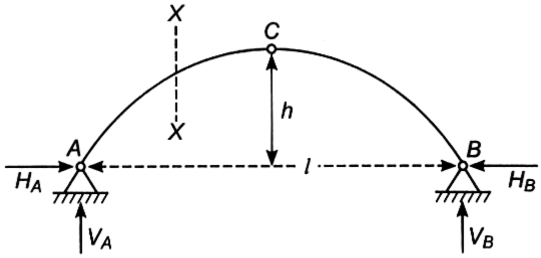
$$\text{Rise in temperature } \delta = \left(\frac{\ell^2 + 4h^2}{4h}\right)\alpha \,.\, t$$

No stresses are produced in a three hinged arch due to temperature change alone. As rise of the arch change due to temperature change, horizontal reaction also changes.

Also, no temperature stress ⇒ no change in bending moment of arch.

$$\underbrace{M_{arch}}_{\text{Constant}} = \underbrace{M_{beam}}_{\text{Constant}} - \underbrace{Hy}_{\substack{\text{have to be}\\ \text{constant as}\\ y\uparrow \text{ So } H\downarrow}}$$

Two Hinged Arches
It is a statically indeterminate structure.
Number of unknowns = 4
Equation of static equilibrium = 3



Two hinged arches

$\therefore$ Degree of static indeterminacy = 4 − 3 = 1

Analysis of Two Hinged Arches

- V_A and V_B can be determined by taking moments about either hinge.
- Horizontal reaction at each support may be determined from the condition that the horizontal displacement of either hinge with respect to other is zero.

Actual bending moment at any section X = Beam moment at X − H moment at X i.e.

$$M_x = M - Hy$$

Total strain energy stored by arch

$$U = \int \frac{M_x^2 ds}{2EI}$$

From Castigliano's first theorem

$$\frac{\partial U_i}{\partial F} = \delta$$

So, $$\frac{\partial U_i}{\partial H} = \delta = 0$$

(No horizontal displacement)

$$U_i = \int \frac{(M - Hy)^2 ds}{2EI}$$

$$\frac{\partial U_i}{\partial H} = \int \frac{2(M - Hy)(-y)ds}{2EI}$$

$$= 0$$

or $$\int \frac{Myds}{EI} - H\int \frac{y^2 ds}{EI} = 0$$

or $$H = \int \frac{\frac{M \cdot yds}{EI}}{\int \frac{y^2 ds}{EI}}$$

where δ = displacement
H = horizontal reaction
M_x= moment at any section X
EI = flexural rigidity

Some Standard Results Regarding Arch

Two hinged parabolic arch

- Two hinged parabolic arch subjected to udl throughout $H = \frac{wL^2}{8h}$
- Udl over left half on span, $H = \frac{wL^2}{16h}$
- A concentrated load w at crown

$$H = \frac{25wL}{128h}$$

- A concentrated load w at a distance x from left end

$$H = \frac{5}{8} \cdot \frac{w.x}{hL^3}(L-x)(L^2+Lx-x^2)$$

Two hinged semi-circular arch

- Two hinged semi-circular arch subjected to udl throughout

$$H = \frac{4}{3} \cdot \frac{wR}{\pi}$$

- Two hinged semi-circular arch subjected to concentrated load w at crown

$$H = \frac{w}{\pi}$$

- A concentrated load w at radius vector α with horizontal

$$H = \frac{w}{\pi}\sin^2\alpha$$

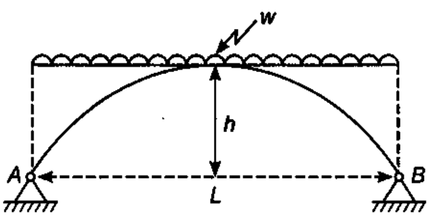
- For a series of concentrated loads at radius vector α with the horizontal

$$H = \sum \frac{w}{\pi}\sin^2\alpha$$

Example 44. A symmetrical 2 hinged parabolic arch of rise r and L is supported at its end on pin at the same level. A load w which uniformly distributed horizontally covers the whole span, the value of the horizontal thrust is

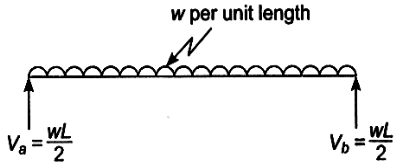
(A) $\frac{wL}{4r}$ (B) $\frac{wL}{6r}$ (C) $\frac{wL}{8r}$ (D) $\frac{wL}{16r}$

Soln. (C)



Horizontal thrust H = $\int \frac{\frac{Myds}{EI}}{\frac{y^2ds}{EI}}$

Where, M = beam moment



$\therefore \quad M = \frac{wL}{2}x - \frac{wx^2}{2}$

and $\quad y = \frac{4hx}{L^2}(L - x)$

Therefore, H $= \dfrac{\int_{x=0}^{x=L}\left(\frac{wx}{2} - \frac{wx^2}{2}\right)\frac{4hx}{L^2}(L-x)\frac{ds}{EI}}{\int_{x=0}^{x=L}\left(\frac{4hx(L-x)}{L^2}\right)^2\frac{ds}{EI}}$

$$= \frac{wL^2 \int \frac{x^2(L-x)^2 ds}{EI}}{8h\int \frac{x^2(L-x)^2 ds}{EI}}$$

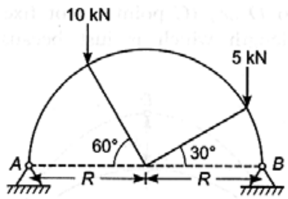
$$= \frac{wL^2}{8h}$$

or $\quad H = \frac{WL}{8r}$

as $\quad W = wL$

and $\quad h = r$

Example 45. A two hinged semicircular arch is loaded as shown in figure, the horizontal thrust at each support is



(A) $\frac{15}{\pi}$kN (B) $\frac{10}{\pi}$kN (C) $\frac{9}{11}$kN (D) $\frac{8.75}{\pi}$kN

Soln. (D)

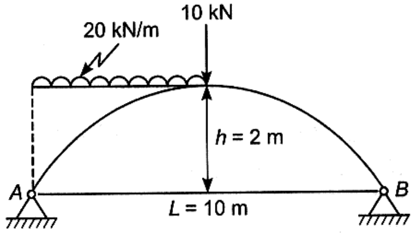
Horizontal thrust H $= \Sigma \frac{W}{\pi}\sin^2 \alpha$

$$= \frac{10}{\pi}\sin^2 60^\circ + \frac{5}{\pi}\sin^2 30^\circ$$

$$= \frac{30}{4\pi} + \frac{5}{4\pi}$$

$$= \frac{8.75}{\pi}\text{kN}$$

Example 46. A two hinged parabolic arch of span 10 m and rise 2 m is uniformly loaded with udl of 20 kN/m over the left half of the span and a concentrated load of 10 kN at crown. The horizontal thrust at each support will be.



(A) 9.76 kN (B) 62.50 kN (C) 72.26 kN (D) 52.74 kN

Soln. (C)

Horizontal thrust due to udl at the left of span

$$H_1 = \frac{wL^2}{16h} = \frac{20\times 10^2}{16\times 2} = 62.5\,\text{kN}$$

Horizontal thrust due to point load at the crown

$$H_2 = \frac{25wL}{128h} = \frac{25\times 10\times 10}{128\times 2}$$

$$= 9.67\text{kN}$$

Thus, total horizontal thrust $H = H_1 + H_2$

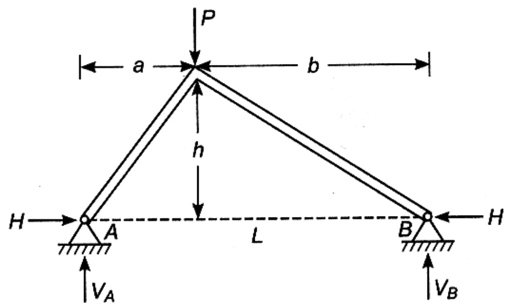
$= 62.5 + 9.76$

$= 72.26$ kN

Rigid joint C is basically moment resistant joint, however for the given loading, there is no bending moments generated at any section.

Linear Arch

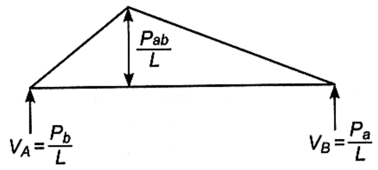
If the vertical loading acting on the arch is composed of concentrated loads, the corresponding free bending moment diagram will also be linear. Such funicular arches are called linear arches. Loads are resisted only by compression, no shear, no bending.



Linear Arch

Horizontal thrust H = $\dfrac{\text{free bending moment } M_{\circ(x)}}{y(x)}$

Beam moment diagram



BMD of linear arch

Thus, $H = \frac{\left(\frac{P_{ab}}{L}\right)}{h}$ $\left\{\text{from } H = \frac{M_{\circ(x)}}{y(x)} \text{ here, } = a\right\}$

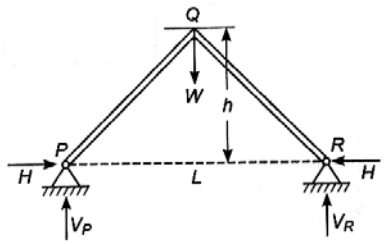
$$H = \frac{P_{ab}}{Lh}$$

where $P_{ab} = \alpha$ system of load round ab

Example 47. A symmetric frame PQR consists of two inclined members PQ and QR connected at Q with a rigid joint and hinged at P and R. The horizontal length PR is l. If a weight w is suspended at Q, the bending moment Q is

(A) $\frac{wl}{2}$ (B) $\frac{wl}{4}$ (C) $\frac{wl}{8}$ (D) zero

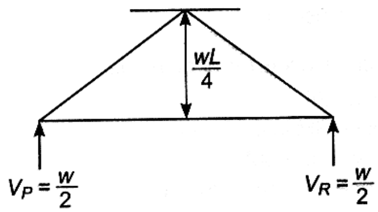
Soln. (D)



It is a case of two-hinged linear arch.
Horizontal thrust,

$$H = \frac{\text{Beam moment } M_{0(x)}}{y(x)}$$

Beam moment diagram



at $\left(x = \frac{L}{2}\right)$

$$H = \frac{\left(\frac{wL}{4}\right)}{h}$$

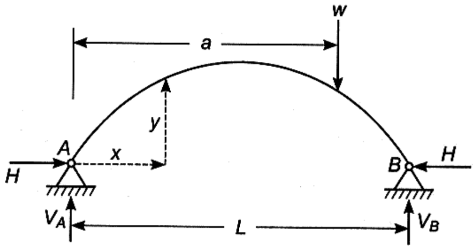
or $$H = \frac{wL}{4h}$$

Actual bending moment at Q = Beam moment Q − H moment at Q

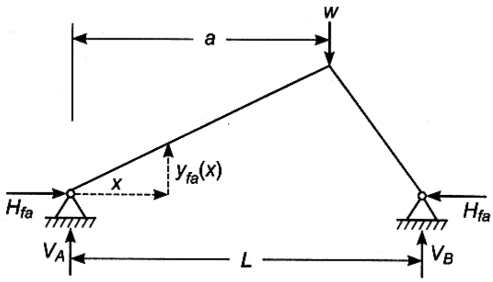
$$= \frac{wL}{4} - \frac{wL}{4h} \cdot h = 0$$

Eddys Theorem

- It states that "the bending moment at any section of an arch is equal to the vertical intercept (the difference in elevation) between the point of the centre-line of given arch and the corresponding point on the funicular (linear) arch having the same span and horizontal thrust for a given system of loads".



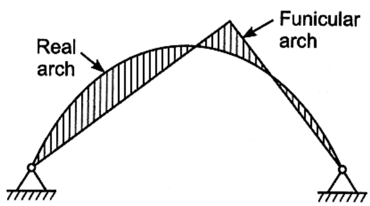
Two hinged arch (Real arch)



Linear (Funicular) arch

As, it is known that the shape of the linear arch follows the shape of the free bending diagram.

- On combing real arch and funicular arch



Free bending diagram of real and funicular arch

$$M(X) = H\left[y_{fa}(x) - y(X)\right]$$

The ordinate between the linear arch and the actual arch gives the bending moment. This is known as Eddy's theorem.

Real arch may be two-hinged or three-hinged.

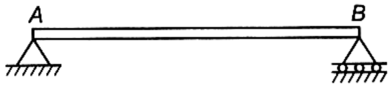
INFLUENCE LINE DIAGRAM

An influence line diagram the variation of a function (such as the shear felt in a structure member) at a specific point on a beam or truss caused by a unit load placed at any point along the structure. Due to static load acting on a structure, various internal forces are developed at each section of the structure. And values of these internal forces (shear, moment) are constant at any particular section.
Now, if load starts moving on the structure, then the values of internal forces also starts changing at each section. Thus the variation of the shear, bending moment, axial forces and support reactions at a specific point in the member due to moving concentrated or distributed load are represented by influence the diagram.

ILD for Beam
The following principle can be utilised to analyse the influence line diagram in beams.
When designing a beam, it is necessary to design for the scenarios causing the maximum expected reactions, shears, moments within the structure members in order to ensure that no member will fail during the life of the structure.

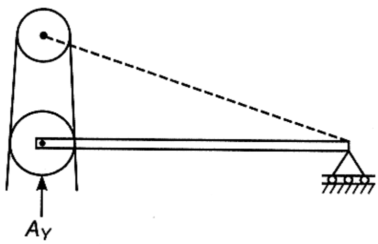
Qualitative ILD
Qualitative means only shape with no ordinate values. The Muller-Breslau principle can be utilised to draw qualitative influence line, which are directly proportional to the actual influence line.
The principle state that ILD for a function (reactions, shear force, bending moment, axial force) is to the same scale as the deflected shape of the beam when the beam is acted upon by the function.

Explanation of Qualitative ILD
In order to draw the deflected shape, the capacity of the beam to resist the applied function must be removed so that beam can deflect when the function applied.
e.g., ILD for vertical reaction at A



Influence line diagram

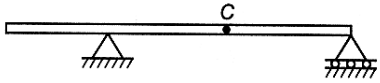
For deflected shape corresponding to vertical reaction at A, remove the pin at A, since the beam must still resist horizontal force at A so provide a roller guide.



ILD of roller guide

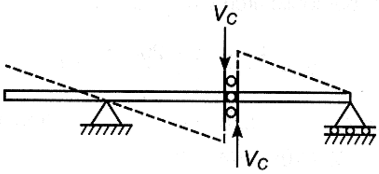
On applying the vertical reaction AY at A, beam deflects to the dashed position. Which represents the general shape of ILD the reaction at A.

ILD for Shear at C



Influence line diagram

For deflected shape corresponding to shear force at C, we have to provide a shear release in the form of roller guide at C.

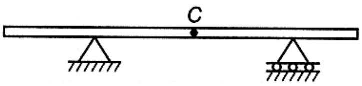


ILD of roller guide

On applying a positive shear force at C, beam deflects to the dashed position, which represents the general shape of ILD for shear at C.

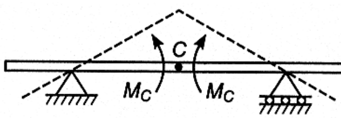
ILD for Bending Moment at C

The FBD of ILD for bending moment at C is shown below.



Influence line diagram

For deflected shape corresponding to bending moment at C, we have to provide moment release in the form of an internal hinge at C.

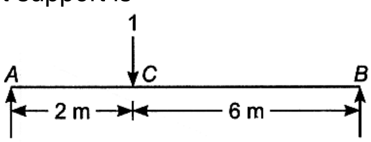


Deflected bending moment

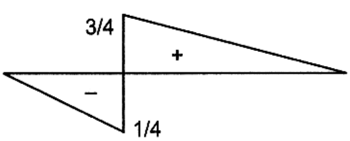
On applying positive moment M_C at C, beam deflects to the dashed position, which represents the general shape of ILD for M_C.

- In drawing ILD, we will consider unit moving load. So, maximum ordinate of ILD will be unity.

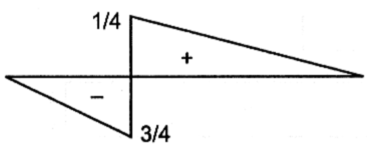
Example 48. The unit load passes over a span of 8 m. The ILD for SF at a section 2m from left support is

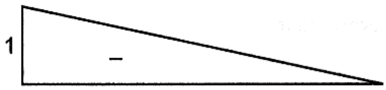


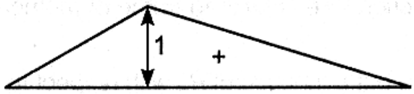
(A)



(B)



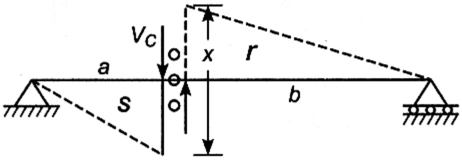
(C) 

(D) 

Soln. (A)

ILD for SF at c

Provide a roller guide at C and then apply V_C.

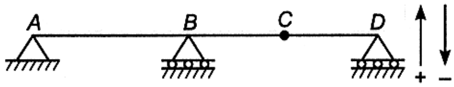


as we know, we take unit moving load, thus when moving unit load will be at point C, then total SF at V_C i.e.,

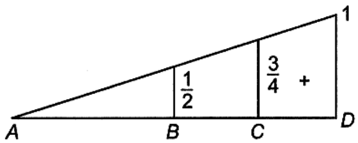
$$x = 1 = \frac{a}{L} + \frac{b}{L}$$

where, a = 2, b = 6, L = 8 in this problem

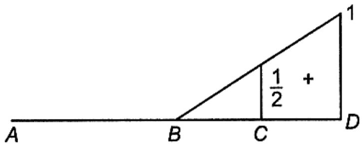
Example 49. For the continuous beam as shown in figure, the ILD for support reaction at D is best represented as



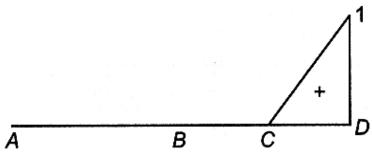
(A)



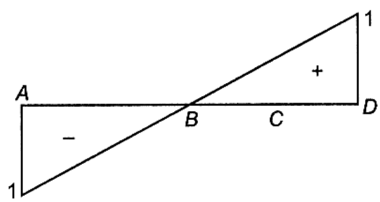
(B)



(C)

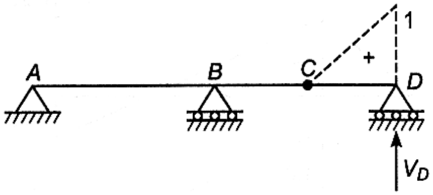


(D)

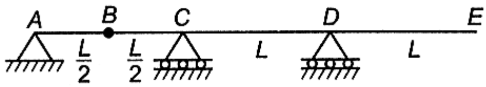


Soln. (C)

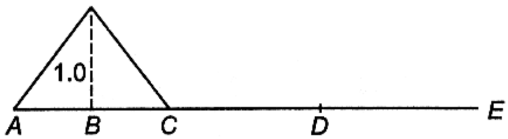
ILD for reaction at D can be drawn by providing unit displacement in the direction of reaction and then deflected shape of beam will represent ILD for V_D. On applying V_D, beam will deflect from hinge C. There will be no deflection in part A to C.



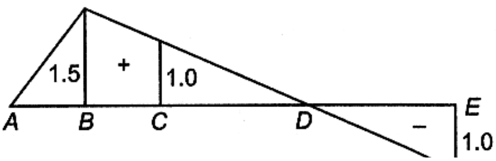
Example 50. The influence line R_C for the beam shown in figure will be as in



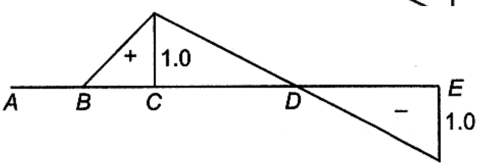
(A)



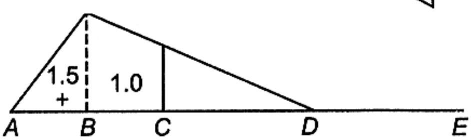
(B)



(C)

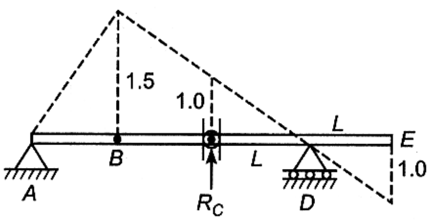


(D)

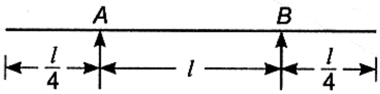


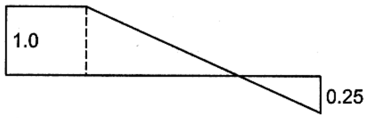
Soln. (B)

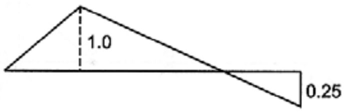
ILD for R_C provide a roller guide at C and then deflected shape corresponding to R_C will represent ILD for R_C ordinate at B and E can be find out by similar triangles as ordinate at C unity.

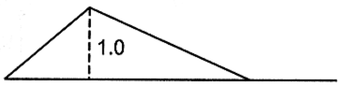


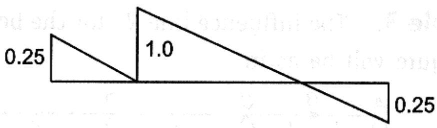
Example 51. A beam with cantilevered ends is shown in the given figure. Which one of the following diagrams represent the ILD for shear just to right of support A?



(A)


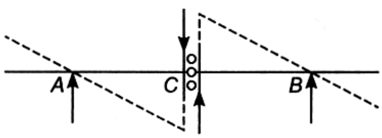
(B)


(C)


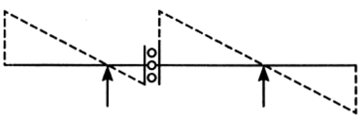
(D)


Soln. (D)

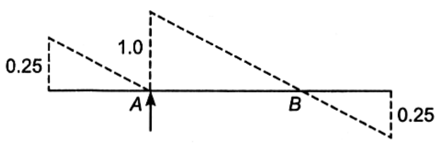
Let us drawing ILD for SF at any section C



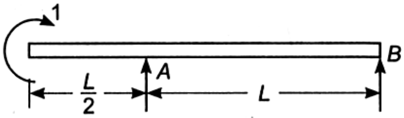
Now, as point C shifts right of A deflected shape will become as



So, ILD for SF just right of A

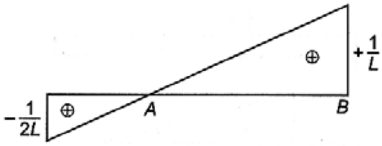


Example 52. A simply supported beam with an overhang is traversed by a unit concentrated moment from left to right as shown below.

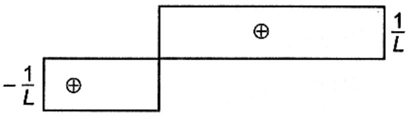


The influence line for reaction at B is given by

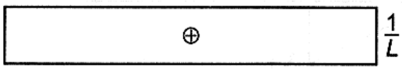
(A)



(B)



(C)

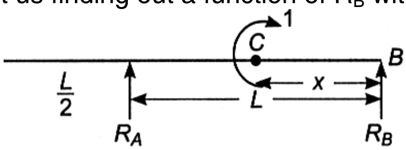


(D) Zero everywhere

Soln. (C)

As we know ILD for any function is the variation of that function at section to section when unit load (force or moment) moves on the straight.

So, let us finding out a function of R_B with respect to the distance moved by unit moment



$R_A + R_B = 0$ ...(i)

taking moments about C = 0

i.e.,

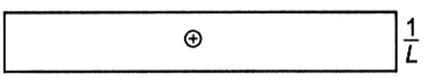
$R_A(L-x)+1=R_B \cdot x$ ...(ii)

Solving Eqs. (i) and Eq. (ii) for R_B

$R_B = \frac{1}{L}$, (which is constant)

Therefore, value of R_B remains constant irrespective of moving unit moment position.

ILD for R_B



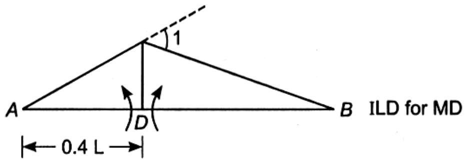
Quantitative ILD

In this section we will discuss the exact values of FSF or BM at any section on beam due to any real load system. ILD for any concentrated load w, can be drawn multiplying ordinates by Considering the example given here.

Example 53. A load W is moving from left to right support on a simply supported beam AB of span L. The maximum bending moment at 0.4L from the left support is

(A) 0.16 wL (B) 0.20 wL (C) 0.24 wL (D) 0.25 wL

Soln. (C)



Ordinate of $M_D = \frac{1\cdot(0.4L)(0.6L)}{L}$ $\left\{\text{from } M = \frac{W_{ab}}{L}\right\}$

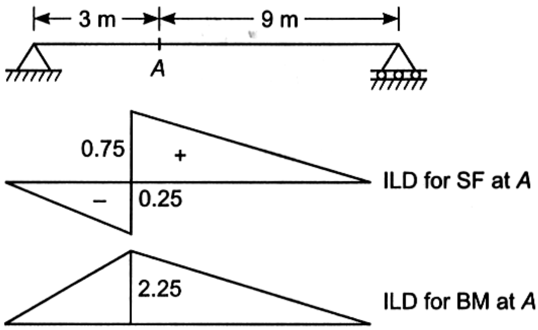
$= 0.24L$

Thus maximum bending moment due to load w

$= (\text{ILD ordinate at D}) \times w$

$= 0.24wL$

Example 54. The given figures shows a beam with its influence line for shear force and bending moment at section A, the values of the SF and BM at section A due to a concentrated load of 20 kN placed at mid span will be

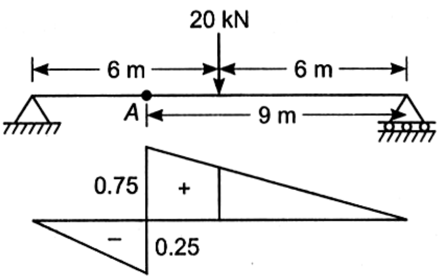


(A) 0.75 kN and 2.25 kN (B) 5 kN and 7 kN

(C) 7.5 kN and 10 kN (D) 10 kN and 30 kN

Soln. (D)

Load 20 kN is at mid section of span



SF at A due to 20 kN $= (\text{ordinate of ILD under load 20 kN}) \times 20\text{ kN}$

$$= \left(\frac{6}{9} \times 0.75\right) \times 20\text{ kN} = 10\text{ kN}$$

BM at A due to 20 kN = (ordinate of ILD under load 20 kN) × 20

$$= \left(\frac{6}{9} \times 2.75\right) \times 20\,\text{kN} = 30\ \text{kN-m}$$

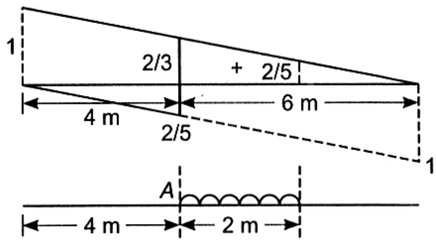
In above problem ILD at section A is given, ordinates at different-different locations simply shows values of SF/BM at section A, when unit load moves. Let the unit load is at mid section then ordinate of ILD at mid section will be value of SF/BM at section A. So for a load w, value of SF/BM at section A will be equal to ordinate of ILD at A due to unit load multiplied by w.

Example 55. A uniformly distributed live load of 40 kN/m of length 2 m moves on a girder of span 10 m. The maximum positive shear force and at a section 4 m from the left and will be

(A) 40 kN (B) 42.67 kN (C) 45 kN (D) 50 kN

Soln. (B)

ILD for SF at section 4 m from left end



For maximum value of Positive SF

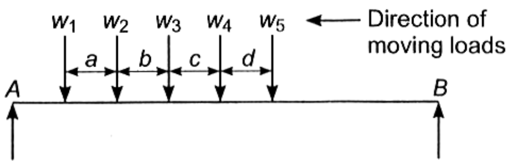
Load position should as

Maximum negative SF = Load intensity × area of ILD covered by load

$$= 40 \times \frac{\left(\frac{2}{3} + \frac{2}{5}\right) \times 2}{2} = 42.67\,\text{kN}$$

Series of Rolling Loads

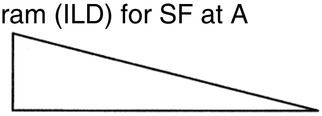
A series of loads w_1, w_2, w_3, w_4, w_5 pass over a simply supported span is shown in figure



Rolling loads acting on beam AB

Maximum End Shear at A

Consider a Influence Line Diagram (ILD) for SF at A



At supports, shear force = Reaction at support For case I When w_1 is exactly at A, the reaction at point a from the load W_1.

R_{a1} = reaction at A (SF)

For case II When w_1 is rolled off and w_2 is exactly at A, the reaction at point a from the load W_2

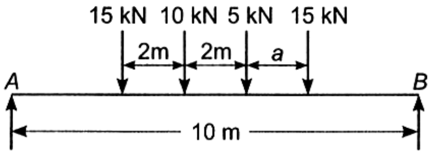
R_{a2} = reaction at A (SF)

for $R_{a1} > R_{a2}$

$$\frac{\text{Load rolled off}}{\text{Successive wheel space}} > \frac{\text{Remaining loads}}{\text{Span length}}$$

i,e.,
$$\frac{w_1}{a} > \frac{(w_2 + w_3 + w_4 + w_5)}{L}$$

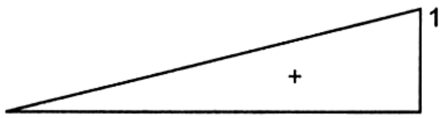
Example 56. A series of rolling loads moves over a simply supported span as shown in figure below. What will be the condition for maximum end shear at support B?



(A) a = 5 m (B) a > 5 m (C) a < 5 m (D) a = 6 m

Soln. (C)

ILD for SF at B

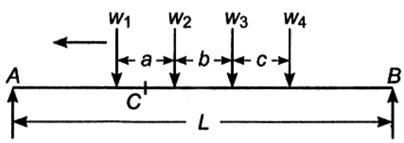


First rolled wheel load = 15 kN

Thus for maximum end shear at B

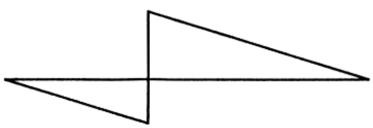
$$\frac{15}{a} > \frac{5+10+15}{10} \text{ or } a < 5 \text{ m}$$

Maximum Shear Force at Given Section



Shear force

Let given section is C; draw ILD for SF at c



ILD for SF

Again, S_{C1} = SF at C with w_2 load at C

S_{C2} = SF at C with w_3 load at C

For $S_{C1} > S_{C2}$

$$\Rightarrow \text{if } \frac{\text{Load rolled off the section C}}{\text{Succeeding wheel space}} > \frac{\text{Sum of all loads}}{\text{Span}}$$

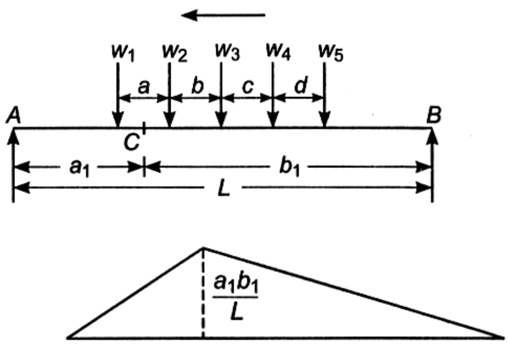
i.e.,
$$\frac{w_2}{b} > \left(\frac{w_1 + w_2 + w_3 + w_4}{\ell}\right)$$

Maximum Bending Moment at a Given Section

Let's consider the following BMD to find out the maximum bending moment at a particular given section.

Let given section is C.

ILD for BM at C



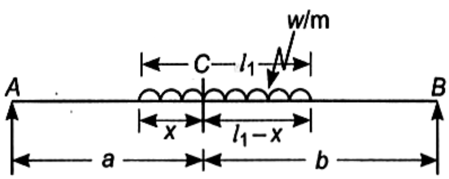
Influence line diagram

For Equilibrium of Forces

Average load on AC = Average load on CB

For udl Shorter than Span

Consider the following span AB of length (a + b) and here the udl works only on length l_1, i.e, l < (a + b), then from the equilibrium of the forces.



udl on a span

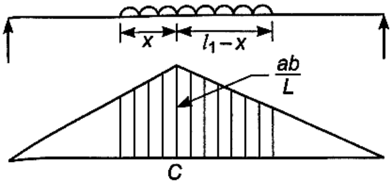
Average load on AC = Average load on CB

$$\frac{wx}{a} = \frac{w(l_1 - x)}{b}$$

$$\frac{b}{a}x = l_1 - x$$

or
$$x = \frac{al_1}{l}$$

For this portion of udl on left of section C, bending moment will be maximum at C.



Value of maximum BM at C = load intensity × area of ILD order load.

- When length of udl is more than span length, then for maximum bending moment at section the span is fully loaded.

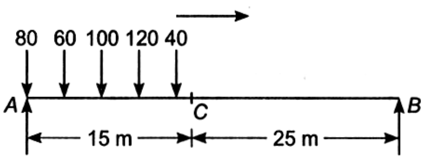
For Moving Wheel Loads

Maximum BM at given section C occurs when average load on left side minus average load on right-side changes sign as point load passes over the section. These can be explained by considering the following examples.

Example 57. Five concentrated loads 40 kN, 120 kN, 100 kN, 60 kN, 80 kN spaced at equal distance of 3 m between them cross from left to right of a SS beam of span 40 m with the 40 kN load leading. Maximum bending moment at section C 15 m from A occurs when

(A) 120 kN load is at C (B) 100 kN load is at C
(C) 60 kN load is at C (D) 80 kN load is at C

Soln. (B)



Let the difference between average load on right-side of C and average load on left side of C is X.

$$X = \left(\frac{40}{25}\right) - \left(\frac{120 + 100 + 60 + 80}{15}\right)$$

$= -12.8$ (negative)

Load 120 kN crossec C

$$X = \left(\frac{40 + 120}{25}\right) - \left(\frac{100 + 60 + 80}{15}\right)$$

$= -9.6$ (negative)

Load 100 kN crosses C

$$X = \left(\frac{40 + 120 + 100}{25}\right) - \left(\frac{60 + 80}{15}\right)$$

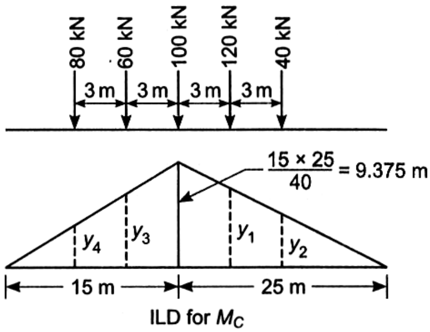
$= 1.067$ (positive)

So, as 100 kN load crosses section C, X changes its sign, therefore, for maximum value of BM at C 100 kN load is at C.

Example 58. In the previous example, what will be the value of maximum moment at section C?

(A) 2876.5 kN-m (B) 3112.5 kN-m (C) 3271.5 kN-m (D) 3476.8 kN-m

Soln. (B)



Maximum BM at C = Loads × (Respective ordinates of ILD)

= $40y_2 + 120y_1 + 100 \times 9.375 + 60y_3 + 80y_4$

y_1, y_2, y_3, y_4 can obtained by similar triangular.

Thus, maximum BM at C

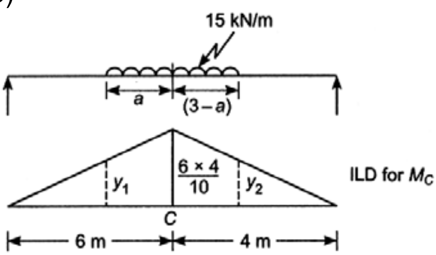
= (40 × 7.125) + (120 × 8.25) + (100 × 9.375) + (60 × 7.5) + (80 × 5.625)

= 3112.5 kN-m

Example 59. A udl of 15 kN/m and length 3 m rolls over a simply supported span of 10 m length. The maximum bending moment at a section 4 m from right end will be

(A) 70.8 kNm (B) 82.2 kNm (C) 90.6 kNm (D) 100 kN

Soln. (C)



Average load on AC = Average load on CB

$$\frac{wa}{6} = \frac{w(3-a)}{4}$$

or $$a = \frac{9}{5} m$$

Maximum BM at C = Load intensity × Area of ILD under load.

$$= 15 \times \left\{\frac{1}{2}(2.4 + 1.84) \times 1.4 + \frac{1}{2}(2.4 + 1.44) \times 1.6\right\}$$

= 90.6 kN-m

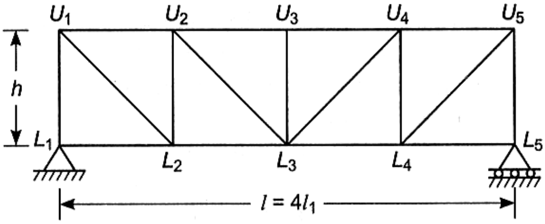
ILD for Truss

Influence lines for forces in truss members can be constructed in much the same manners as those for the beam. The basic principle of an influence line is made use of, which indicates the variation of the force in any member of the truss as the unit load moves across the truss.

Types of Truss Members

A general truss consists of four types of members.

- Bottom chord members
- Top chord members
- Vertical members
- Diagonal member



Truss

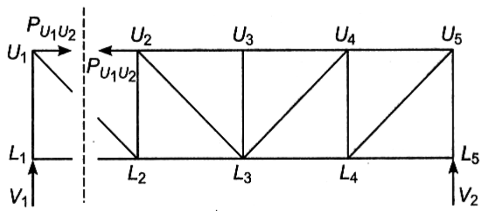
ILD means variation of shear, moment, axial force or support reaction as unit load moves over span.

A truss structure is composed of bar members, which resists load either by compression or by tension. Therefore, here ILD will be variation of axial forces in various truss members as unit load moves from L_1 to L_2.

ILD for Top Chord Members

In first step, pass a imaginary vertical section that cut the desired top member.

Member U_1 U_2

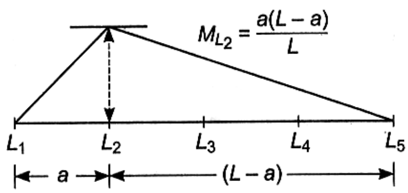


Influence line diagram for top chord

For member U_1 U_2, (opposite bottom chord joint is L_2)

$$P_{U_1U_2} = \frac{M_{L_2}}{h}$$

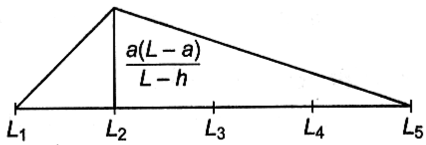
where, M_{L_2} is bending moment at L_2 due to unit load considering span L_1L_5 as simply supported.



ILD

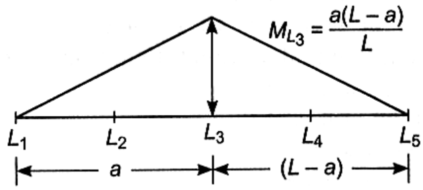
So, ILD for $P_{U_1U_2} = \frac{\text{ILD for } M_{L_2}}{h}$

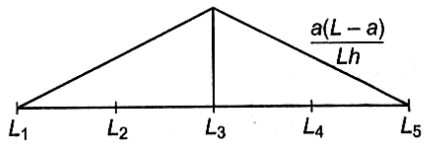
i.e.,



ILD for U_2U_3 (opposite bottom chord joint L_3)

$$P_{U_2U_3} = \frac{M_{L_3}}{h}$$

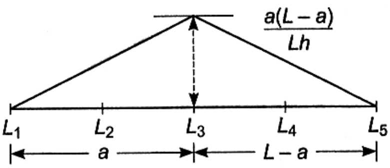




ILD for $P_{U_2U_3}$

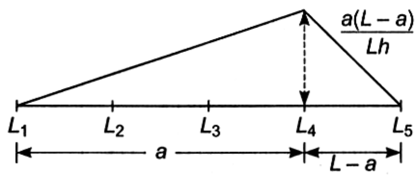
ILD for U_3U_4 (opposite bottom chord joint L_3)
thus ILD for U_3U_4 will be same as for U_2U_3

$$P_{U_3U_4} = \frac{M_{L_3}}{h}$$



ILD for U_4U_5 (opposite bottom chord joint L_4)

$$P_{U_4U_5} = \frac{M_{L_4}}{h}$$



All top chord members have compressive force so we draw ILD for them as positive (conventional)

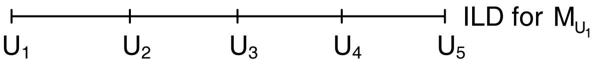
ILD for Bottom Chord Members

We will use similar procedure, as used in top chord member. Here we will find opposite top chord joint.

ILD for L_1L_2 (opposite top chord joint U_1)

$$P_{L_1L_2} = \frac{M_{U_1}}{h}$$

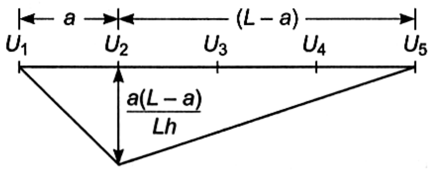
Where, M_{U_1} is bending moment at U_1, due to unit load, considering span U_1U_5 as simply supported.



$$M_{U_1} = \frac{a\,(L-\alpha)}{L} = 0 \qquad \{\therefore\ a = 0\}$$

ILD for L_2L_3 (opposite top chord joint U_2)

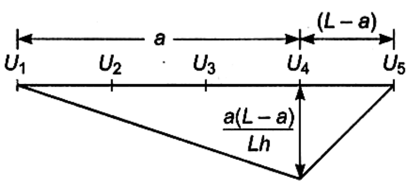
$$P_{L_2L_3} = \frac{M_{U_2}}{h}$$



Influence line diagram

ILD for L_3L_4 (opposite top chord joint U_4)

$$P_{L_3L_4} = \frac{M_{U_4}}{h}$$



Influence line diagram

ILD for L_4L_5 (opposite top chord joint U_5)

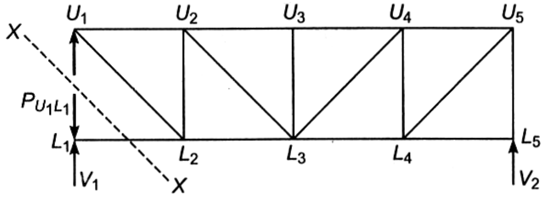
$$P_{L_4L_5} = \frac{M_{U_5}}{h}$$

Since, $a = 0 \therefore M_{U_5} = 0$

All bottom members have tensile forces, so draw ILD as below axis (as convention)

ILD for Vertical Members

ILD for member U_1L_1 Pass a section through member U_1L_1 that cut it into two parts.



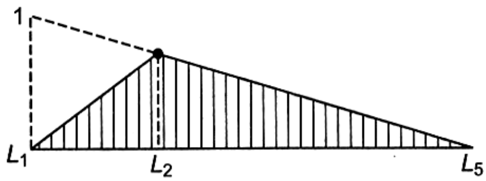
ILD for vertical member

Here, our objective is to find out vertical member force $P_{U_1L_1}$ as unit load moves from L_1 and L_5.

Let unit load is at joint L_1. In this case member U_1L_1 will take no load and support reaction $V_1 = 1$ (obvious understanding). As load moves right of L_1, member U_1L_1 start taking some load to balance support reaction V_1. Now, consider unit load be anywhere on right side of joint L_2.

$$P_{L_1U_1} = V_1 \quad \text{(compressive)}$$

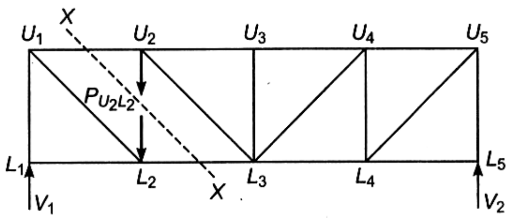
Thus ILD for $P_{L_1U_1}$ will ILD of reaction V_1 when load is right of L_2.



Influence line diagram

we will consider only right part of L_2 of ILD for V_1.

ILD for member U_2L_2



Influence line diagram

When unit load is right of L_3

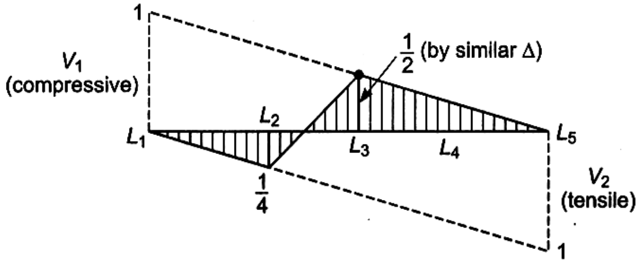
consider left part $P_{U_2L_2} = V_1$ (compressive)

when unit load is left of L_2

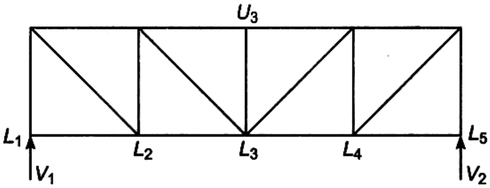
consider right part $P_{U_2L_{22}} = V_2$ (tensile)

For nature of $P_{U_2L_2}$ see the FBD of above truss.

Compressive force are drawn above the axis as convention.



ILD for U_3L_3

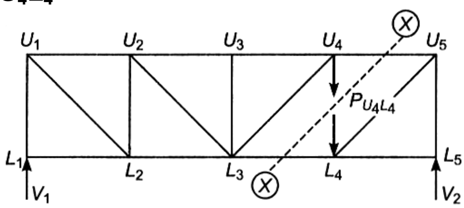


Influence line diagram

Now from analysis of truss, if at a joint three member passes out of which two are collinear, then force in third member will be zero, provided that there is no external load at that joint.

Hence, $P_{U_3L_3} = 0$

ILD for U_4L_4



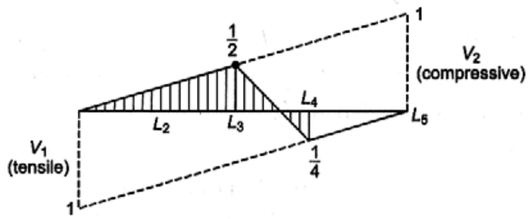
Influence line diagram

when unit load is left of L_3
consider right part,

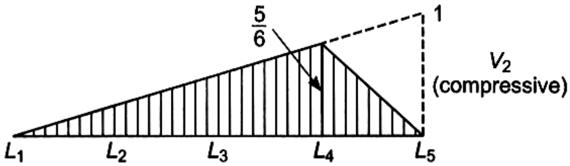
$$P_{U_4L_4} = V_2 \qquad \text{(compressive)}$$

when unit load is right of L_4
consider left part

$$P_{U_4L_4} = V_1 \qquad \text{(tensile)}$$



Influence line diagram

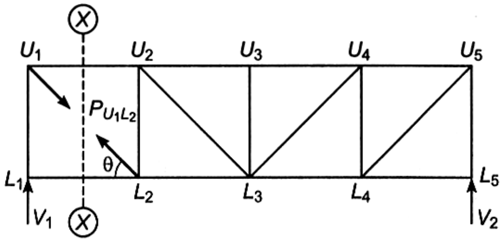


Influence line diagram

ILD for Diagonal Members

ILD for member U_1U_2

Pass a imaginary vertical section, which cut the diagonal member U_1L_2 in two parts.



Influence line diagram

As unit load is right side of L_2.

Consider left part of section X – X

$$P_{U_1L_2}\ \sin\theta = V_1$$

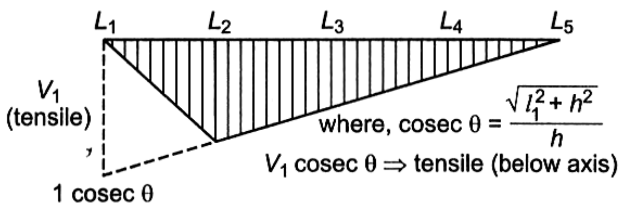
$\Rightarrow$ $$P_{V_1L_2} = V_1 \operatorname{cosec}\ \theta \quad \text{(tensile)}$$

As unit load is exactly at L_1. There will be zero force in U_1L_2

because $(V_1 = 1)$

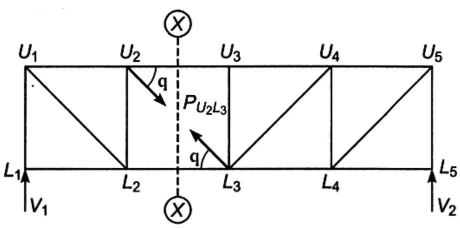
Thus for unit load between L_1 to L_2

$P_{U_1L_2}$ will vary from 0 to V_1 cosec θ



Influence line diagram

ILD for U_2U_3



Influence line diagram

- When unit load is on right side of joint L_3 considering left part of section X – X

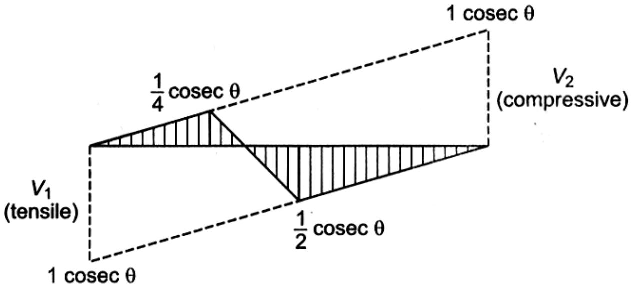
  $P_{U_2L_3}\sin\theta = V_1$ (tensile)

  or $P_{U_2L_3} = V_1 \text{ cosec}\,\theta$ (tensile)

- When unit load is on left side of joint L_2 considering right part of section X – X

  $P_{U_2L_3}\sin\theta = V_2$ (compressive)

  or $P_{U_2L_3} = V_2 \text{ cosec}\,\theta$

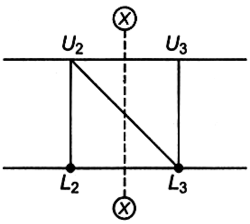


Influence line diagram

In the same way ILD for U_4L_3 and U_5L_4 can be drawn.

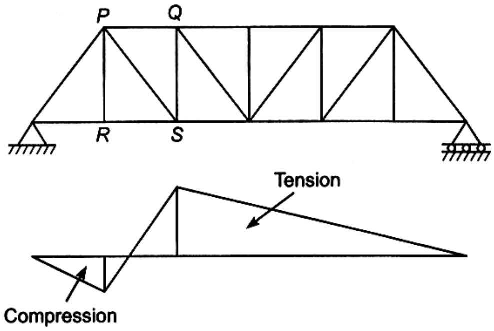
Influence Line Diagram : At a Glance

- Make a imaginary cut section through the desired member.
- See right and left joint of that section to about how many member to know about side of the unit load.
- If unit load is on right of that section, consider left part of left joint (L_2) see below figure.
- If unit load is on left of the section, consider right part of right joint (L_3), see below figure.



- Top chord members are under compression, so ILD will be drawn above axis (only conventional).
- Bottom chord member are under pull tension, so ILD will be drawn below axis (only conventional).
- Observe carefully the exact middle vertical member.
- For top chord member, see just opposite connecting bottom chord joint (for U_2U_3, joint L_3).
- For bottom chord member, see just opposite connecting top chord joint (for L_2L_3, joint U_2)

Example 60. The Influence Line Diagram (ILD) shown is for the member.

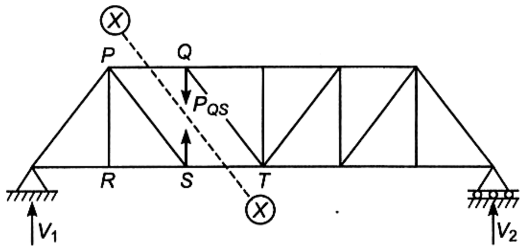


(A) PQ (B) PS (C) RS (D) QS

Soln. (A)

- Member PQ is a top chord member, which will remain in compression. So above ILD is not possible for PQ.
- Member RS is a bottom chord member, which will remain in tension. So above ILD is not possible for RS.

Member QS

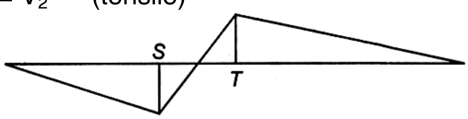


When unit load is right of T,
consider right part of section X – X

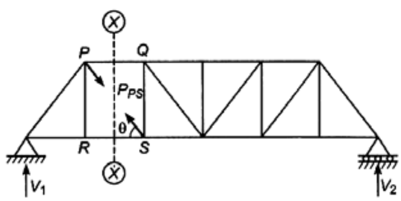
$P_{QS} = V_1$ (compressive) from equilibrium

When unit load is left of S,
consider right part of section X – X

$P_{QS} = V_2$ (tensile)



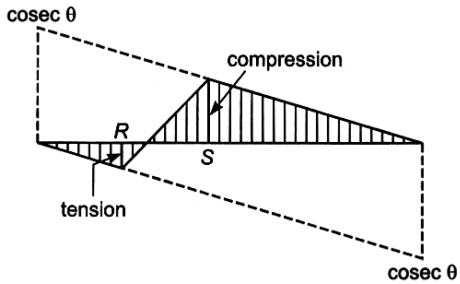
Member PR



Influence line diagram

Opposite joint of cut-section R and S.
for load right-side of S, $P_{PS} = V_1$ (tensile). cosec θ

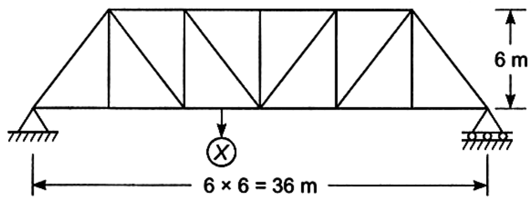
for load left-side of R, $P_{PS} = V_2$ (compressive). cosec θ



ILD for member PS or QS

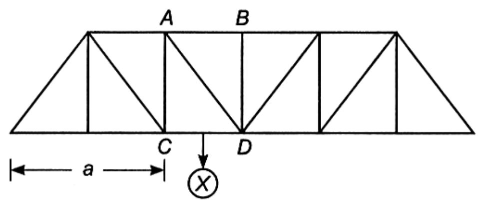
Hence, the given ILD may be for member PS or QS.
But only R and S joint are given in figure, so it should be ILD for PS, as ILD for member QS contains joints S and T
Sign convention for the compression is taken (-ve) and for tension (+ve), which makes no difference.

Example 61. What is the maximum ordinate for influence line for the force in the member X?



(A) 1.0 (B) 1.33 (C) 1.50 (D) 2.50

Soln. (B)



Opposite joint of bottom chord member X is top chord joint A.

Therefore, $P_X = \dfrac{M_A}{h}$

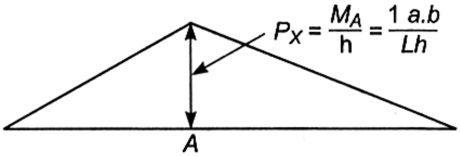
where, $M_A = \dfrac{1 \cdot a \cdot b}{L}$ $\left\{\text{from } M = \dfrac{Wab}{L}\right\}$

and a = 12 m
b = 24 m
L = 36 m
h = 6 m

Thus, $P_X = \dfrac{M_A}{h}$

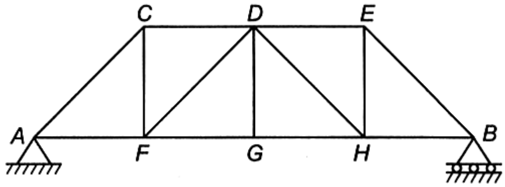
$$= \frac{1 \cdot a \cdot b}{Lh}$$

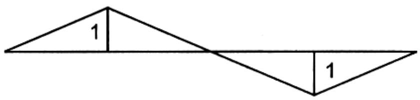
ILD for P_X



Ordinate of $P_X = \dfrac{1 \times 12 \times 24}{36 \times 6} = 1.33$

Example 62. Which one of the following diagrams represent the ILD for force in member DG?



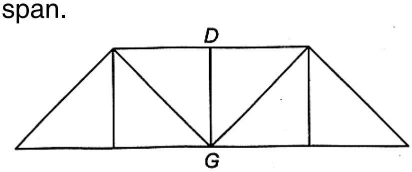
(A)


(B)


(C)

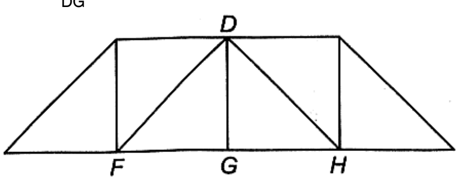
(D)


Soln. (D)
From synopsis, if at a joint three members are passing out of which two are collinear then force in non-collinear member is always zero provided with no external load at that joint.
The above statement will be true if truss have following shape, as Unit load moves on bottom span.

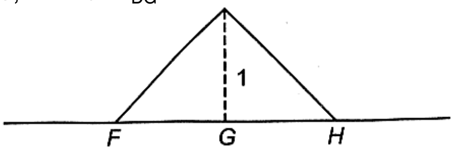


But, in the given truss member DG is connected in the following pattern.
When unit load is exactly at G.

$F_{DG} = 1$

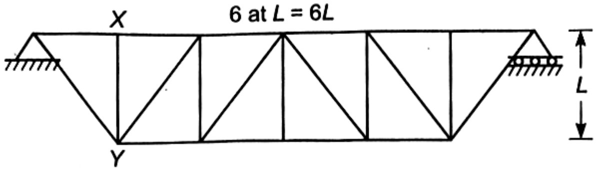


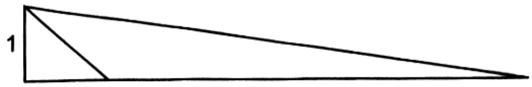
There will be no member force in DG for unit load position before F and after H.
Therefore, ILD for F_{DG}

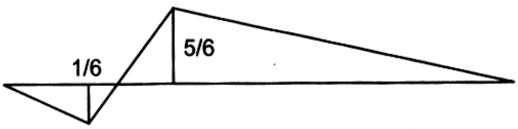


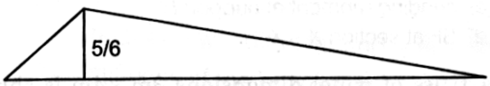
In previous example, we used through type truss in which bottom chord members received load.
All things of analysis remain same if we use, Deck type truss except here top chord joints receive loads.

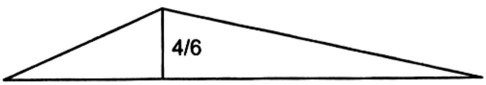
Example 63. For the shown deck type truss ILD for member XY will be



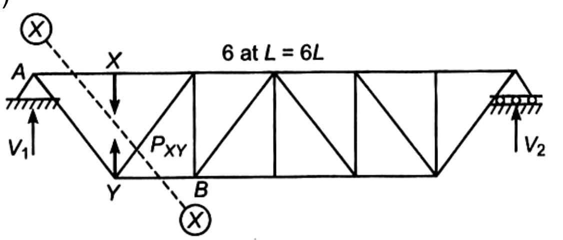
(A)


(B)


(C)


(D)


Soln. (C)



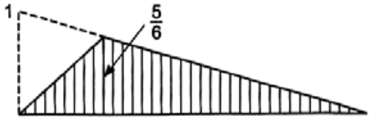
As load is received by top chord joints.
So, we will consider joints A and X not Y and B.
on either side of section X - X for unit load on right of joint X

$P_{XY} = V_1$ (compressive)

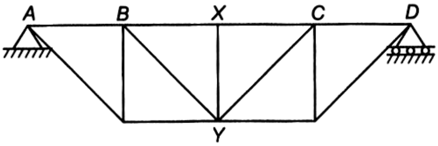
when unit load is exactly at A

$P_{XY} = 0$ as $V_1 = 1$

ILD for P_{XY}

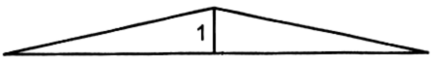


Example 64. The influence line diagram for the force in member XY of the truss shown in figure will be

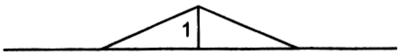


(A)

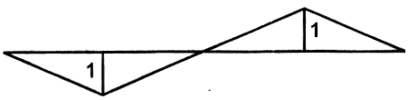
(B)



(C)



(D)



Soln. (C)

As unit load moves over top chord members, so member force XY will not be zero.

when unit load is exactly at X

$F_{XY} = 1$ (compressive)

Member XY is considered as secondary member

BASIC CONCEPT OF MATRIX METHOD

In general a structure have a number of nodes, therefore our objective is here to write force and displacement relations in matrix form, to simplify a problem. Generally, we use the matrix method for simplicity in calculation.

Terminology Related Matrix

Flexibility

It is displacement caused by unit force

i.e., $f = \frac{\Delta}{P}$ (where P = 1 unit)

where Δ = displacement

Stiffness

It is the force required for unit displacement.

i.e., $K = \frac{P}{\Delta}$ (where Δ = 1 unit)

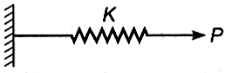
Stiffness simply means resistance against deformation. Stiffer, the member lesser will be its deformation or more force has to be apply for unit deformation. Flexibility (f) and stiffness (K) can be related as

$f \cdot K = 1$

i.e., flexibility and stiffness are reciprocal of each other.

Relation between Flexibility and Stiffness

Using spring deformation



Spring deformation

If we apply pull P on a spring of stiffness (K), then deformation (Δ) can be given by relation.

$P = K \cdot \Delta$

remember above relation.

For flexibility, put $K = \frac{1}{f}$ in above relation

$P = \frac{1}{f} \cdot \Delta$

where f = flexibility

Formation of Matrix

Stiffness (K_{ij}) for multidegree of freedom structural element

$P_i = K_{ij} \cdot \Delta_j$

i $\rightarrow$ force position

j $\rightarrow$ displacement position

e.g., $P_1 = K_{12} \cdot \Delta_2$ implies force is applied at coordinate I and displacement is required at coordinate 2

or $\quad K_{12} = \frac{P_1}{\Delta_2} \quad$ (where, $\Delta_2 = 1$)

Stiffness (K_{12}) is the force required in the member at node (1) to produce unit displacement at node (2)

Flexibility (f_{ij}) for multidegree of freedom structural element

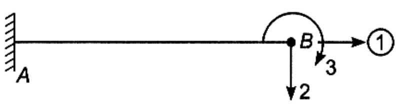
$\Delta_i = f_{ij} \cdot P_j$

Flexibility (f_{ij}) is the displacement at coordinate (i) due to a unit force at coordinate (j)

e.g., $\quad f_{12} = \frac{\Delta_1}{P_2}$

Let a structural member AB, which is fixed at end A and end B can have three types of displacements.

Axial displacement Δ_1 at coordinate (1)



Beam with one fixed end

On applying axial pull P_1 at coordinate (1), the displacement at coordinate 1 is given by equation

$\frac{P_1 L}{AE} = \Delta_1$

Now, from definition $f_{11} = \frac{\Delta_1}{P_1}$

i.e., $f_{11} = \frac{L}{AE}$ Axial flexibility

also $K_{11} = \frac{P_1}{\Delta_1}$

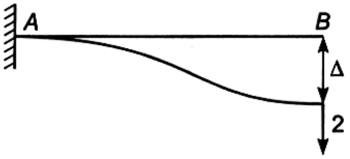
i.e., $K_{11} = \frac{AE}{L}$ Axial stiffness

Transverse displacement Δ_2 at coordinate (2)

(a) End A-fixed

For displacement Δ along coordinate (2)

force $P_2 = \frac{12EI\Delta_2}{L^3}$ (Remember)



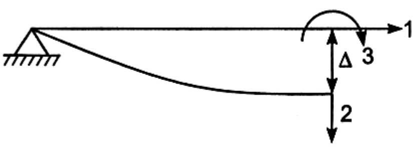
Beam with one fixed end

Note that there is no displacement along coordinate L and (3).
Now, from definition

$$f_{22} = \frac{\Delta_2}{P_2} = \frac{L^3}{12EI} \quad \text{(transverse flexibility)}$$

and $$K_{22} = \frac{P_2}{\Delta_2} = \frac{12EI}{L^3} \quad \text{(transverse stiffness)}$$

(b) End A-hinged



Hinged end

$$P_2 = \frac{3EI}{L^3} \cdot \Delta_2$$

$$f_{22} = \frac{L^3}{3EI} \quad \text{(transverse flexibility)}$$

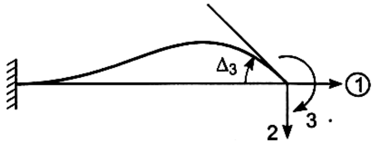
$$K_{22} = \frac{3EI}{L^3} \quad \text{(transverse stiffness)}$$

Bending displacement Δ_3 (rotation) at coordinate (3)

(a) End A-fixed

Only rotational displacement (Δ_3) at coordinate (3) no displacement at coordinate (1) and (2)

$$P_3 = \frac{4EI}{L} \cdot \Delta_3$$

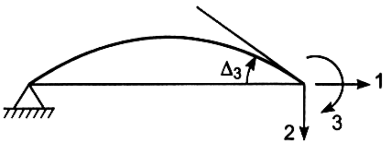


Rotating Hinged end

Thus, $f_{33} = \frac{\Delta_3}{P_3} = \frac{L}{4EI}$ (flexural flexibility)

and $K_{33} = \frac{4EI}{L}$ (flexural stiffness)

(b) End A-hinged



End A-hinged

$$P_3 = \frac{3EI}{L} \cdot \Delta_3$$

$$f_{33} = \frac{L}{3EI}$$

and $K_{33} = \frac{3EI}{L}$

Term EI is known as flexural rigidity. So more the flexural rigidity, more will be stiffness of member.
i.e., In formula of K, (EI) term will be numerator always.

Stiffness Matrix

In the finite element method for the numerical solution of elliptic partial differential equations, the stiffness matrix represents the system of linear equations that must be solved in order to as certain an approximate solution to the differential equation.

i.e., $P_i = K_{ij} \cdot \Delta_j$

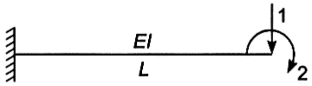
Let j = 1, i = 1, 2, 3 ……n

or $P_i = K_{i1} \cdot \Delta_1$ implies if a unit displacement is given at coordinate (1) without any displacement at other coordinates, then forces required at coordinates 1, 2, 3 … n may be represented by K_{i1} {where, i = 1, 2, 3, ….n}

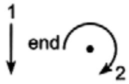
$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_n \end{bmatrix}_{1\times n} = \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1j} & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2j} & \cdots & K_{2n} \\ K_{31} & K_{32} & \cdots & K_{3j} & \cdots & K_{3n} \\ \vdots & & & & & \vdots \\ K_{n1} & K_{n2} & \cdots & K_{nj} & \cdots & K_{nn} \end{bmatrix}_{n\times n} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \vdots \\ \Delta_n \end{bmatrix}_{n\times 1}$$

In above K-matrix (which is a square matrix), the jth column represents the forces at coordinate 1, 2, 3…n due to unit displacement given at coordinate j, without any displacement at other coordinates.

Example 65. Generate the stiffness matrix for a cantilevered beam.



Soln. Total displacement coordinates are two

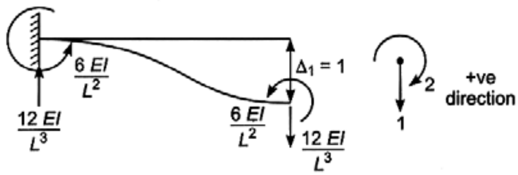


i.e., number of nodes are two
[K] matrix will be of order 2 × 2

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

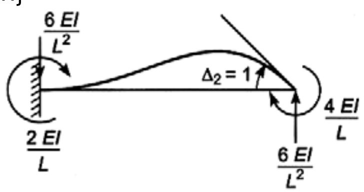
we will formulate [K] matrix, columnwise i.e., $\begin{bmatrix} K_{12} \\ K_{22} \end{bmatrix}$ or $\begin{bmatrix} K_{11} \\ K_{21} \end{bmatrix}$

(a) Giving unit displacement at coordinate (1) i.e., $\Delta_1 = 1$ finding out K_{11} and K_{21} {first column}



thus, $K_{11} = \frac{12EI}{L^3}, K_{12} = \frac{-6EI}{L^2}$

(b) Unit displacement at coordinate (2) i.e., $\Delta_2 = 1$ finding out K_{21} and K_{22} {second column}



$$K_{21} = \frac{-6EI}{L^2}, K_{22} = \frac{4EI}{L}$$

Therefore, $[K]_{2\times2} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$

1. K-matrix's leading diagonal is always positive
2. K-matrix is always symmetric i.e., $K_{ij} = K_{ji}$.
3. Remember results corresponding to $\Delta_1 = 1$ and $\Delta_2 = 1$ with deflected shape diagram.

Calculation of Flexibility Matrix

Flexibility [f] matrix is inverse of [K] matrix.

So, $[f] = [K]^{-1}$

$$= \begin{bmatrix} \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}^{-1}$$

Inverse of a matrix can be find out as

$$[A]^{-1} = \frac{1}{|A|} \text{adj}[A]$$

where, adj [A] = transpose of cofactor matrix

$$[K] = \begin{bmatrix} \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

Cofactor $C_{11} = (-1)^{1+1} \cdot \frac{4EI}{L} = \frac{4EI}{L}$

$$C_{12} = (-1)^{1+2} \cdot \left(\frac{-6EI}{L^2}\right) = \frac{+6EI}{L^2}$$

$$C_{11} = (-1)^{2+1} \cdot \left(\frac{-6EI}{L^2}\right) = \frac{6EI}{L^2}$$

$$C_{22} = (-1)^{2+2} \cdot \left(\frac{12EI}{L^3}\right) = \frac{12EI}{L^3}$$

$\therefore$ Cofactor matrix [C] = $\begin{bmatrix} \frac{4EI}{L} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{12EI}{L^3} \end{bmatrix}$

Transpose of [C] matrix = adj [K] = $\begin{bmatrix} \frac{4EI}{L} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{12EI}{L^3} \end{bmatrix}$

determinant K = $|K| = \frac{48 \cdot (EI)^2}{L^4} - \frac{36\,(EI)^2}{L^4} = \frac{12\,(EI)^2}{L^4}$

Therefore, flexibility matrix $[f] = \frac{\text{adj}[K]}{|K|}$

$$= \frac{L^4}{12(EI)^2} \begin{bmatrix} \frac{4EI}{L} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{12EI}{L^3} \end{bmatrix}$$

$$[f] = \begin{bmatrix} \frac{L^3}{3EI} & \frac{L^2}{2EI} \\ \frac{L^2}{2EI} & \frac{L}{EI} \end{bmatrix}$$

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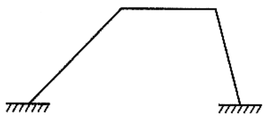
ASSIGNMENT – 1

Duration : 45 Min. **Max. Marks : 30**

Q1 to Q6 carry one mark each

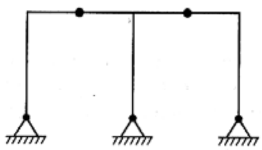
Direction (Q.1 – 18) : Calculate the degree of static indeterminacy of following structure.

1. Frame



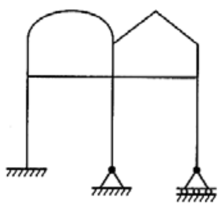
(A) 0 (B) 2 (C) 3 (D) 4

2. Frame



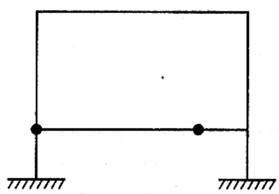
(A) 1 (B) 2 (C) 3 (D) 4

3. Frame



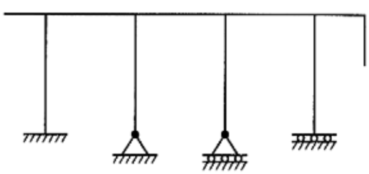
(A) 3 (B) 6 (C) 9 (D) 12

4. Frame



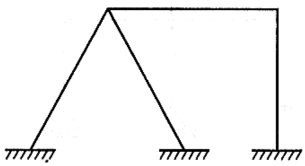
(A) 0 (B) 1 (C) 2 (D) 3

5. Frame



(A) 2 (B) 3 (C) 5 (D) 6

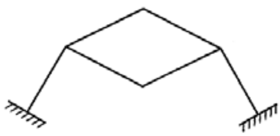
6. Frame



(A) 3 (B) 6 (C) 9 (D) 12

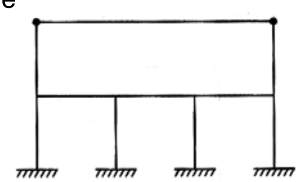
Q7 to Q18 carry two marks each

7. Frame



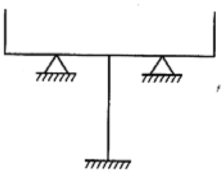
(A) 3 (B) 5 (C) 6 (D) 7

8. Frame



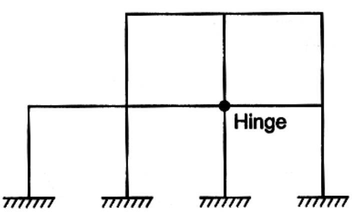
(A) 10 (B) 11 (C) 12 (D) 14

9. Frame



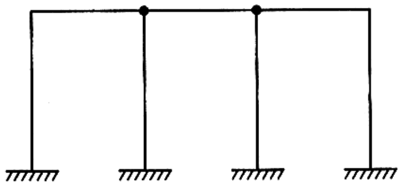
(A) 4 (B) 3 (C) 2 (D) zero

10. Frame



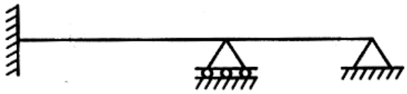
(A) 10 (B) 11 (C) 12 (D) 15

11. Frame



(A) 4 (B) 5 (C) 7 (D) 9

12. Beam

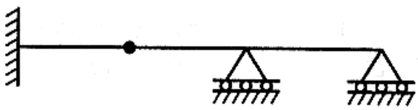


(A) 3 (B) 6 (C) 9 (D) 0

13. Beam

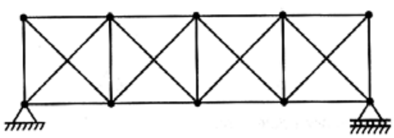
(A) 0 (B) 1 (C) 2 (D) 3

14. Frame



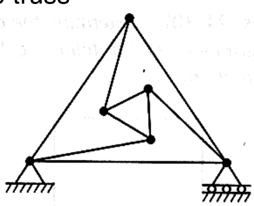
(A) 0 (B) 1 (C) 2 (D) 3

15. Plane truss



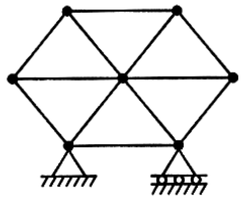
(A) 1 (B) 2 (C) 3 (D) 4

16. Plane truss



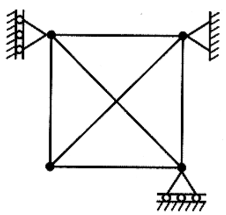
(A) 0 (B) 1 (C) 2 (D) 3

17. Plane truss



(A) 0 (B) 1 (C) 2 (D) 3

18. Truss



(A) unstable, $D_S = 1$
(B) stable, $D_S = 2$
(C) stable, $D_S = 0$
(D) unstable, $D_S = 2$

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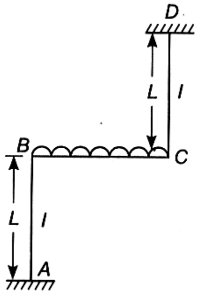
TEST PAPER – 1

Duration : 30 Min. **Max. Marks : 25**

Q1 to Q5 carry one mark each

1. A plane frame is loaded as shown in the figure. The rotations are indicated as θ_B and θ_C and sway is indicated by symbol Δ. For the given frame which one of the statement is correct?



(A) $\theta_B = \theta_C$; Δ is absent
(B) $\theta_B = -\theta_C$; Δ is absent
(C) $\theta_B = \theta_C$; Δ is present
(D) $\theta_B = -\theta_C$; Δ is present

2. The portal frame shown in Fig. I was analysed and the final column moments were found to be shown as shown in the fig. II. The value of P is

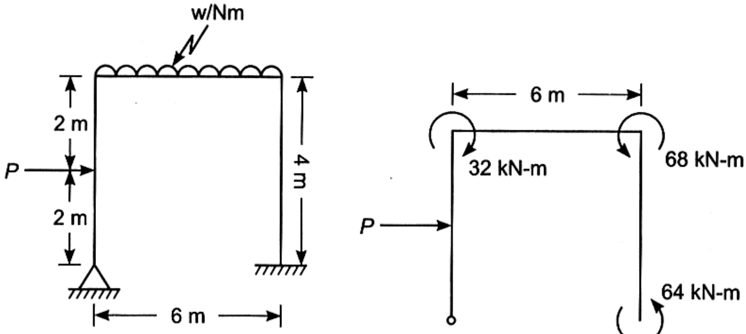


Fig. I Fig. II

(A) 25 kN
(B) 41 kN
(C) 45 kN
(D) 50 kN

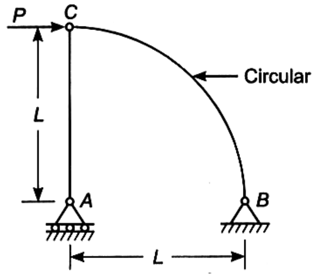
3. The internal forces at any cross-section of an arch are
(A) SF only
(B) SF and BM only
(C) normal thrust only
(D) SF, BM and normal thrust all

4. A three hinged symmetrical arch carries a udl over the entire span, then the section of the arch is subjected to
(A) SF only
(B) SF and BM
(C) SF and normal thrust
(D) normal thrust only

5. The effect of arching a beam is
(A) to reduce SF
(B) to reduce BM in the span
(C) to increase the BM
(D) to reduce the normal thrust

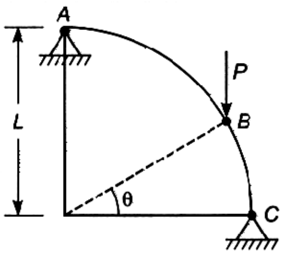
Q6 to Q15 carry two marks each

6. Reaction at support B of the structure is



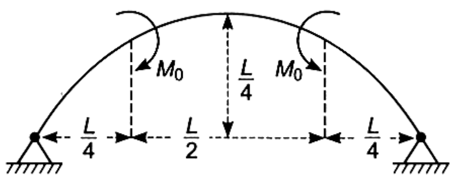
(A) P (B) $P\sqrt{2}$ (C) $\frac{P}{\sqrt{2}}$ (D) $\frac{P}{2}$

7. A three hinged arch shown in the figure is quarter of a circle. If the vertical and horizontal components of reaction at A are equal, the value of θ is



(A) 60° (B) 45° (C) 30° (D) None of the these

8. A two hinged parabolic arch carries two concentrated moments as shown in figure. The resultant at left support will be

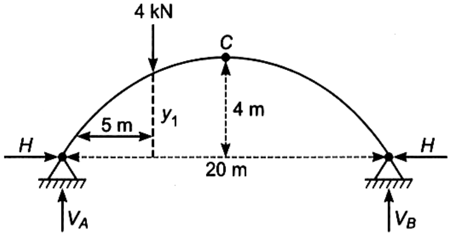


(A) 0 (B) M_0 (C) $\frac{55}{32}M_0$ (D) $\frac{55}{16}M_0$

9. Due to certain temperature increase, the rise of the three hinged arch increased by 1%. The horizontal thrust will then
(A) increase by 1% (B) decrease by 1%
(C) No change (D) None of these

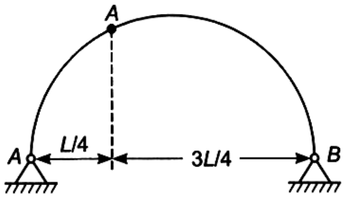
10. A three hinged arch consisting of two quadrantal parts AC and CB of radii R_1 and R_2. The arch carries a concentrated load w on the crown. The horizontal thrust is
(A) $\frac{w}{4}$ (B) $\frac{w}{2}$ (C) $\frac{3w}{4}$ (D) w

11. For the three hinged parabolic arch, shown the value of the horizontal reaction is



(A) 1 kN (B) 5 kN (C) 4 kN (D) 2.5 kN

12. For the semi-circular two-hinged arch shown in the figure below, a moment of 50 kN-m applied at B produces a displacement of 0.5 cm at A. If a concentrated load of 10 kN is applied at A, the rotation at B in the arch will be

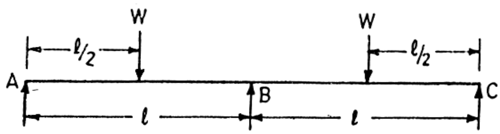


(A) 0.1 (B) 0.01 (C) 0.001 (D) 0.0001

13. A circular three pinned arch of span 40 m and a rise of 8 m is hinged at the crown and springing. It carries a horizontal load of 100 kN per vertical metre on the left side. The horizontal thrust at the right springing will be
(A) 200 kN (B) 400 kN (C) 600 kN (D) 800 kN

14. Which one of the following statement is correct? Linear arch is one which represents
(A) centre line of an arch
(B) variation of bending moment
(C) thrust line
(D) variation of shear force

15. For the continuous beam (EI constant) loaded as shown in the figure given, the moment at 'B' is



(A) 0.75 times free moment at mid–span of AB
(B) same as the free moment at mid–span of AB
(C) 1.50 times free moment at mid–span of AB
(D) 2.00 times free moment at mid–span of AB

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